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Stability of Magnetohydrodynamic Micropolar Channel Flow with Thermal Radiation, Internal Heating, and Species Transport

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ABSTRACT

This study investigates the transport characteristics and stability of micropolar fluid flow in a channel under the combined effects of thermal radiation, internal heat generation or absorption, and chemical reaction. The governing equations for momentum, microrotation, energy, and species concentration are transformed into a coupled system of nonlinear ordinary differential equations through appropriate similarity transformations and nondimensionalisation. The resulting equations are solved using the collocation weighted residual method showcasing the effects of the Prandtl number, thermal radiation parameter, heat-function parameter, and micropolar parameter under heat-generation and heat-absorption conditions. The results indicate that heat generation enhances the temperature distribution, whereas heat absorption produces the opposite effect. Increasing the thermal radiation parameter reduces the temperature profile within the investigated parameter range. The stability analysis further indicates that variations in thermal radiation and the heat-function parameter influence the predicted onset of instability, while the micropolar parameter exhibits a stabilising tendency under the conditions considered. These findings highlight the coupled influence of thermal transport and fluid microstructure on the behaviour of micropolar channel flow. The study provides a mathematical framework for examining the interaction between thermal effects, species transport, and micropolar characteristics in magnetohydrodynamic flow systems.

INTRODUCTION

Micropolar fluid mechanics extends classical fluid theory by incorporating the local rotational motion and microstructural characteristics of fluid elements. The foundational theories of Eringen (1964, 1966) established a mathematical framework for describing fluids whose microscopic structures influence their macroscopic behaviour. Ariman et al. (1973) further reviewed the development and applications of microcontinuum fluid mechanics. Unlike classical Newtonian fluid models, micropolar fluid theory accounts for additional degrees of freedom associated with microrotation and the intrinsic angular momentum of fluid particles. These characteristics make the theory particularly relevant to the investigation of complex transport phenomena involving momentum transfer, heat transfer, and hydrodynamic stability.

Hydrodynamic stability describes the response of a basic flow to small disturbances and determines whether such perturbations decay or grow with time. Understanding this behaviour is important in predicting flow transitions and identifying the conditions under which a fluid system maintains its equilibrium state. Several researchers have investigated the stability characteristics of micropolar fluids under different physical conditions. Ahmed (1976) examined the stability of a micropolar fluid layer heated from below, while Dhiman et al. (2011) investigated convective stability using a variational approach. Reena and Rana (2009) studied thermosolutal convection in a porous micropolar fluid, whereas Weng et al. (2009) considered stability between rotating concentric cylinders. These investigations established the importance of thermal, mechanical, and microstructural parameters in determining the stability behaviour of micropolar fluid systems.

In addition to these stability considerations, thermal radiation and internal heat generation or absorption play significant roles in modifying the temperature distribution and associated transport characteristics of micropolar fluids. Shamshuddin (2017). Heat and mass transfer on the unsteady MHD flow of chemically reacting micropolar fluid with radiation and Joule heating, while Salawu and Fatumbi (2017) examined dissipative heat transfer under variable electrical conductivity and non-uniform heat generation. In a related investigation, Nwabuzor et al. (2018) studied magnetohydrodynamic micropolar fluid flow in a porous medium under the combined influence of heat generation or absorption and thermal radiation. Their analysis examined the effects of relevant dimensionless parameters on velocity, microrotation, temperature, and concentration distributions, together with the associated skin-friction, Nusselt, and Sherwood numbers. Extending this investigation, Nwabuzor et al. (2019) examined the stability of micropolar fluid flow in a porous medium under the influence of heat function, magnetic field, and thermal radiation. Their findings indicated that the Prandtl number, wave number, radiation, and heat-function parameters influence the onset of stability, while the electromagnetic and micropolar parameters exhibit different stabilising tendencies.

More recently, Srinivasacharya and Barman (2024) investigated the linear stability of micropolar fluid flow in a vertical channel, providing further insight into the influence of flow and microstructural characteristics on disturbance development. Subsequently, Barman and Srinivasacharya (2025) examined the effects of thermal radiation and a transverse magnetic field on the stability of magnetohydrodynamic micropolar flow. Although these studies have advanced the understanding of micropolar flow stability under thermal and electromagnetic influences, their respective configurations and investigated parameter combinations differ from the formulation considered in the present study. In particular, the combined examination of thermal radiation, internal heat generation or absorption, and coupled species transport in relation to the stability response of micropolar channel flow remains an important area for further investigation.

Motivated by these considerations, the present study investigates the hydrodynamic stability of micropolar fluid flow in a channel under the combined effects of thermal radiation and internal heat generation or absorption. The mathematical formulation incorporates the coupled momentum, angular momentum (microrotation), energy, and species concentration equations, accounting for the relevant thermal, electromagnetic, diffusive, and chemical reaction effects. Suitable similarity transformations and nondimensionalisation are employed to reduce the governing partial differential equations to a coupled system of nonlinear ordinary differential equations. A disturbance-based stability analysis is subsequently performed to examine the response of the flow to small perturbations and assess the influence of the governing dimensionless parameters on its stability characteristics.

The principal objective of this investigation is to establish how thermal radiation and heat generation or absorption influence the coupled transport characteristics and hydrodynamic stability of micropolar fluid flow in a channel. Particular attention is given to the effects of the relevant Prandtl, Grashof, Schmidt, radiation, heat-function, magnetic, electroconductivity, chemical reaction, and micropolar parameters included in the mathematical model. The findings are expected to provide further theoretical understanding of the interaction between thermal transport, species diffusion, fluid microstructure, and disturbance behaviour in micropolar channel flows.

PHYSICAL FORMULATION OF THE PROBLEM

The present study considers the flow of an incompressible micropolar fluid subject to an imposed transverse magnetic field, thermal radiation, internal heat generation or absorption, and a first-order homogeneous chemical reaction. The fluid motion is described by the velocity components (u) and (v) in the principal-flow and transverse directions, respectively, while (N) denotes the microrotation field. The temperature and concentration fields are represented by (T) and (C), respectively. The model accounts for the coupling between linear momentum and microrotation, together with the thermal and solutal buoyancy contributions. The applied magnetic field introduces an electromagnetic resistance term into the momentum equation. The energy equation incorporates thermal conduction, radiative heat transfer, internal heat generation or absorption, and viscous dissipation, while the species equation accounts for molecular diffusion and first-order chemical reaction. The wall is maintained at a prescribed temperature (T_w), with the ambient temperature denoted by (T_∞), where ($T_w > T_\infty$). A velocity-slip condition is imposed at the wall, and an appropriate microrotation condition is specified to represent the interaction between the fluid microstructure and the bounding surface. The concentration field is governed by the prescribed wall concentration and the ambient concentration. The assumptions of steady laminar flow, incompressibility, and constant fluid properties are adopted, except where temperature-dependent terms are explicitly included. The resulting conservation equations provide the mathematical framework for investigating the coupled momentum, angular momentum, heat-transfer, and species-transport processes. The precise computational domain and its boundary conditions are specified

consistently with the selected channel or boundary-layer geometry.

Governing Equations

The governing equations are derived from the conservation of mass, linear and angular momentum, energy, and chemical species. The formulation accounts for micropolar fluid effects, magnetic force, thermal radiation, temperature-dependent heat generation or absorption, and first-order chemical reaction. Appropriate wall and far-field boundary conditions are imposed. Similarity transformations and nondimensionalisation are then applied to obtain the dimensionless governing equations for subsequent analysis.

$$\frac{U\partial u}{\partial x} + \frac{V\partial u}{\partial y} = (\frac{\mu+K}{\rho})\frac{\partial^2 u}{\partial y^2} + \frac{k}{\rho}\frac{\partial N}{\partial y} - \frac{\sigma B^2}{\rho}u + g\beta_T(T - T_\infty) + g\beta_e(C - C_\infty) - \frac{\sigma_\infty u'}{U} \quad (1)$$

$$u\frac{\partial N}{\partial X} + V\frac{\partial N}{\partial y} = \frac{\epsilon}{\rho j}\frac{\partial^2 N}{\partial y^2} - \frac{K}{\rho j}(\frac{\partial u}{\partial y} + 2N) \quad (2)$$

$$U\frac{\partial T}{\partial x} + V\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho C_p}(T - T_\infty) - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} + \mu(\frac{\partial u}{\partial y})^2 \quad (3)$$

$$U\frac{\partial C}{\partial X} + V\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - K_r(X)(C - C_\infty) \quad (4)$$

Where $K_r(X)$ = Chemical potential, $\frac{Q}{\rho C_p}$ = Heat function, $\frac{1}{\rho C_p}\frac{\partial q}{\partial y}$ = Radiation term, $\mu(\frac{\partial u}{\partial y})^2$ = Viscous dissipation term, $\frac{\sigma_\infty u'}{U}$ = Electroconductivity term

Following Kalidas et al. (2016) the appropriate boundary conditions for the present model are given by

$$\widehat{U} = U_w + N_1(\hat{x})\, v\frac{\partial \hat{u}}{\partial y}\ \check{V} = 0,$$

$$\widehat{N} = -n\frac{\partial \hat{u}}{\partial \hat{y}},\ T = T_w + D_1(\hat{x})\frac{\partial T}{\partial \hat{y}},\ C = C_w \text{ at } \hat{y} = 0 \quad (5)$$

$$\widehat{U} = 0,\ \widehat{N} = 0,\ T = T_\infty \quad C = C_\infty \text{ as } \hat{y} > \infty$$

For the similarity solution to equation (1) to (4) be conquered with the boundary condition in equation (5) the following similarity transformation, dimensionless variable are adopted in the rest of the analysis;

SIMILARITY TRANSFORMATION

$$\varphi = \sqrt{\frac{v}{D}}x.f(n), n = y\sqrt{\frac{D}{v}}\frac{\partial n}{\partial y} = \sqrt{\frac{D}{v}}\frac{\partial^2 n}{\partial y^2} = 0, U = \frac{\partial \varphi}{\partial y} = \sqrt{\frac{v}{D}}x.f'(n)\frac{\partial n}{\partial y}, v = -\frac{\partial \varphi}{\partial y} = -\sqrt{\frac{v}{D}}f(n),$$

$$\frac{\partial u}{\partial x} = \sqrt{\frac{v}{D}}f'(n), \frac{\partial u}{\partial y} = \sqrt{\frac{v}{D}}x.f''(n)\frac{\partial n}{\partial y} + \sqrt{\frac{v}{D}}x.f'(n)\frac{\partial^2 n}{\partial y^2}; \frac{\partial^2 u}{\partial y^2} = \sqrt{\frac{v}{D}}x.f'''(n)\frac{\partial n}{\partial y} + \sqrt{\frac{v}{D}}x.f''(n)\frac{\partial^2 n}{\partial y^2} + \sqrt{\frac{v}{D}}x.f'(n)\frac{\partial^3 n}{\partial y^3},$$

$$h(n) = \frac{N}{D\sqrt{\frac{v}{D}}x}, \frac{\partial N}{\partial y} = \sqrt{\frac{v}{D}}xh'(n)\frac{\partial n}{\partial y}; \frac{\partial^2 N}{\partial y^2} = \sqrt{\frac{v}{D}}xh'(n)\frac{\partial n}{\partial y} + \sqrt{\frac{v}{D}}xh'(n)\frac{\partial^2 n}{\partial y^2}; \frac{\partial N}{\partial x} = \sqrt{\frac{v}{D}}h(n), \theta(n) = \frac{T - T_\infty}{T_w - T_\infty},$$

$$\phi(n) = \frac{C - C_\infty}{C_w - C_\infty}; \frac{\partial T}{\partial x} = \frac{\partial \theta(n)}{\partial x} = \frac{\partial \theta}{\partial n}\frac{\partial n}{\partial x} = \theta'(n).0, \frac{\partial T}{\partial y} = \frac{\partial \theta(n)}{\partial y} = \frac{\partial \theta}{\partial n}\frac{\partial n}{\partial y} = \theta'(n)\sqrt{\frac{D}{v}}; \frac{\partial^2 T}{\partial y^2} = \frac{\partial^2 \theta}{\partial y^2} = \theta''(n)\sqrt{\frac{D}{v}} + \theta'(n).0,$$

$$\frac{\partial C}{\partial x} = \frac{\partial \phi(n)}{\partial x} = \frac{\partial \phi}{\partial n}\frac{\partial n}{\partial x} = \phi'(n).0, \frac{\partial C}{\partial y} = \frac{\partial \phi(n)}{\partial y} = \frac{\partial \phi}{\partial n}\frac{\partial n}{\partial y} = \phi'(n)\sqrt{\frac{D}{v}}; \frac{\partial^2 C}{\partial y^2} = \frac{\partial^2 \phi}{\partial y^2} = \phi''(n)\sqrt{\frac{D}{v}}$$

By using Rosseland approximation q_r takes the form

$$q_r = -\frac{4\sigma_I}{3K_I}\frac{\partial T^4}{\partial y} \quad (6)$$

Where σ_I is the Stefan-Bolyzmann constant and K_I is the mean absorption coefficient. It is assumed that the temperature difference within the flow are sufficiently small such that temperature differences within the flow are sufficiently small such that T^4 can be expressed as a linear function of temperature. This is accomplished by expanding T^4 can be expressed as a linear function of temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher order terms thus

$$T^4 \equiv 4T_\infty^3, T - 3T_\infty^4 \quad (7)$$

Using equation (6) and (7), equation (3) takes the form

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \propto\frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho C_p}(T - T_\infty) + \frac{16\sigma_I T_\infty^3}{\rho C_P K_I}\frac{\partial^2 T}{\partial y^2} + \frac{v}{C_p}(\frac{\partial u}{\partial y})^2 \quad (8)$$

Substituting equation similarity transformation into equation (1), (2), (4) and (8) it gives

$$x\,f'(n)\,f'(n) - \sqrt{\frac{v}{D}}f(n)x\,f''(n) = \frac{\mu+k}{\rho}x(n)\,f'''(n) - \frac{\sigma B^2}{\rho}x\,f'(n) + gB_T\theta(n) + gB_e\emptyset(n) - \frac{\sigma_\alpha}{v}x\,f'(n) \quad (9)$$

$$x\,f'(n)\left(D\sqrt{\frac{D}{v}}\,h(n)\right) - \sqrt{\frac{v}{D}}f(n)\left(Dxh'(n)\right) = \frac{\varepsilon}{\rho j}Dxh''(n) - \frac{k}{\rho j}x\,f''(n) + 2h(n)D\sqrt{\frac{D}{v}}\,x \quad (10)$$

$$-f(n)\theta'(n) = \sqrt{\frac{D}{v}}\alpha\theta''(n) + \frac{Q\theta}{\rho C_p} + \frac{16}{3}\frac{\sigma_I T_\alpha^3}{\rho k C_\rho}\sqrt{\frac{D}{v}} + \frac{\mu}{C_p}x\,f'(n) \quad (11)$$

$$-f(n)\,\phi'(n) = D_m\,\phi''(n)\sqrt{\frac{D}{v}} - K_r(x)\emptyset(n) \quad (12)$$

DIMENSIONAL ANALYSIS

Equation (10) – (12) is transformed in to dimensionless parameters of the Schmidts number, Prandtls number, Grashof number, Modified Grashof number, Radiation parameter, Heat function, Micropolar parameter, Microrotational parameter Electroconductivity and Magnetic term. The dimensionless quantities used are

Substituting dimensional parameters into equations (9) to (12), results

$$k f'''(n) + K h'(n) - f'(n) f'(n) - \sigma f'(n) - f(n)M + f'(n)f(n) + G_r\theta(n) + G_m\emptyset(n) = 0 \quad (13)$$

$$\lambda h''(n) + h'(n)f(n) - k\left(f''(n) + 2h(n)\right) - f'(n)h(n) = 0 \quad (14)$$

$$\theta''(n)(\Pr - R) + f(n)\theta'(n) + Q\theta(n) + \Pr f''(n) f''(n) = 0 \quad (15)$$

$$Sc\,\phi''(n) + f(n)\phi'(n) - K_r\phi(n) = 0 \quad (16)$$

Applying the boundary conditions

$$f' = 0, f = 0, h = -n f'', \theta = 1, \phi = 1 \ at\ n = 0$$

$$f' \to 0, h \to 0, \theta \to 0, \phi \to 0\ as\ n \to \infty \quad (17)$$

From equation (16) the solution is

$$\emptyset(n) = x_0 + x_1 n + x_2 n^2 + x_3 n^3 + x_4 n^4 + x_5 n^5 + x_6 n^6 + x_7 n^7 + x_8 n^8 \quad (18)$$

$$\phi'(n) = x_1 + 2x_2 n + 3x_3 n^2 + 4x_4 n^3 + 5x_5 n^4 + 6x_6 n^5 + 7x_7 n^6 + 8x_8 n^7 \quad (19)$$

$$\phi''(n) = 2x_2 + 6x_3 n + 12x_4 n^2 + 20x_5 n^3 + 30x_6 n^4 + 42x_7 n^5 + 56x_8 n^6 \quad (20)$$

$$f(n) = b_0 + b_1 n + b_2 n^2 + b_3 n^3 + b_4 n^4 + b_5 n^5 + b_6 n^6 + b_7 n^7 + b_8 n^8 \quad (21)$$

Substitute equation (19) – (21) into equation (16) it implies

$$Sc\,(2x_2 + 6x_3 n + 12x_4 n^2 + 20x_5 n^3 + 30x_6 n^4 + 42x_7 n^5 + 56x_8 n^6) + \left(b_0 + b_1 n + b_2 n^2 + b_3 n^3 + b_4 n^4 + b_5 n^5 + b_6 n^6 + b_7 n^7 + b_8 n^8\right)(x_1 + 2x_2 n + 3x_3 n^2 + 4x_4 n^3 + 5x_3 n^4 + 6x_4 n^5 + 7x_5 n^6 + 8x_6 n^7) - k_r(x_0 + x_1 n + x_2 n^2 + x_3 n^3 + x_4 n^4 + x_5 n^5 + x_6 n^6 + x_7 n^7 + x_8 n^8) \quad (22)$$

Taking the coefficients in equation (22)

$$-k_r x_0 + 2Scx_2 + x_1 b_0 \quad (23)$$

$$\left(-k_r x_2 + 12Scx_3 + 2x_2 b_0 + x_1 b_1\right)n \quad (24)$$

$$(-k_r x_2 + 12Scx_4 + 3x_3 b_0 + 2x_2 b_1 + x_1 b_2)n^2 \quad (25)$$

$$(-k_r x_3 + 20Scx_4 + 4x_4 b_0 + 3x_3 b_1 + 2x_2 b_3 + x_1 b_3)n^3 \quad (26)$$

$$(-k_r x_4 + 30Scx_6 + 5x_5 b_0 + 4x_4 b_1 + 3x_3 b_2 + 2x_2 b_3 + x_1 b_4)n^4 \quad (27)$$

$$\left(-k_r x_5 + 42Sc\,x_7 + 6x_6 b_0 + 5x_5 b_1 + 4x_4 b_2 + 3x_3 b_3 + 2x_2 b_4 + x_1 b_5\right)n^5 \quad (28)$$

$$(-k_r x_6 + 56Scx_6 + 7x_7 b_0 + 6x_6 b_1 + 5x_5 b_2 + 4x_4 b_3 + 3x_3 b_4 + 2x_2 b_5 + x_1 b_6)n^6 \quad (29)$$

$$(-k_r x_7 + 8x_8 b_0 + 7x_7 b_1 + 6x_6 b_2 + 5x_5 b_3 + 4x_4 b_4 + 3x_3 b_5 + 2x_2 b_6 + x_1 b_7)n^7 \quad (30)$$

$$(-k_r x_8 + 8x_8 b_1 + 6x_6 b_3 + 5x_5 b_4 + 4x_4 b_5 + 3x_3 b_6 + 2x_2 x_7 + x_1 b_8)n^8 \quad (31)$$

$$(8x_8 b_2 + 7x_7 b_3 + 6x_6 b_4 + 5x_5 b_5 + 4x_4 b_6 + 3x_3 b_7 + 2x_2 b_8)n^9 \quad (32)$$

$$(8x_8 b_3 + 7x_7 b_4 + 6x_6 b_5 + 5x_5 b_6 + 42x_4 b_7 + 3x_3 b_8)n^{10} \quad (33)$$

$$(8x_5 b_4 + 7x_6 b_5 + 6x_6 b_6 + 5x_5 b_7 + 4x_4 b_8)n^{11} \quad (34)$$

$$(8x_6 b_5 + 7x_7 b_6 + 6x_6 b_7 + 5x_5 b_8)n^{12} \quad (35)$$

$$(8x_8 b_6 + 7x_7 b_7 + 6x_6 b_8)n^{13} \quad (36)$$

$$(8x_8 b_7 + 7x_7 b_8)n^{14} \quad (37)$$

$$8x_5 b_8 n^{15} \quad (38)$$

Applying collocation weighted residual method, equations (23 – 38) take the form

$$\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4 \dots\dots\dots\dots\dots\dots\dots\dots\dots \alpha_{15} \quad (39)$$

Sum equation (39) it results

$$\alpha_0 + \alpha_1 n + \alpha_2 n^2 + \alpha_3 n^3 + \alpha_4 n^4 + \alpha_5 n^5 + \alpha_6 n^6 + \alpha_7 n^7 + \dots\dots + \alpha_{14} n^{14} \quad (40)$$

Using the Collocation Weighted Residual

$$\varepsilon_I = -C_{app} + \sum_{k=0}^{\alpha} \alpha_k \phi_k \quad (41)$$

Where ε_I is the internal domain error, C_{app} is the concentration approximate solution, α_k represents the unknown coefficient. ϕ_k, is the independent function $\emptyset_k$. Comes from a complete sequence defined by

$$\emptyset_1 = n(1-n) \quad (42)$$

$$\emptyset_2 = n(1-n)n \quad (43)$$

$$\emptyset_3 = n(1-n)n^2 \quad (44)$$

To get an approximate solution, the internal domain error ε_I must be forced to zero that is

$$0 = -T_{app} + \sum_{k=0}^{\alpha} \alpha_k \phi_k \quad (45)$$

$$T_{app} = \sum_{k=0}^{\alpha} \alpha_k \phi_k \quad (46)$$

For convenience we select only two collocation point that is let us choose $\alpha_0 + \alpha_1 n$ and let

$$C_{app} = \alpha_0(n(1-n) + \alpha_1 n(n(1-n)) \quad (47)$$

$$C_{app} = (n-n^2)\alpha_0 + (n^2-n^3)\alpha_1 \quad (48)$$

$$C^I_{app} = 1-2n, \qquad C^{II}_{app} = -2 \quad (49)$$

$$C_{app} = n^2 - n^3$$

$$C^I_{app} = 2n-3n^2, \qquad C^{II}_{app} = 2-6n \quad (50)$$

Put equations (49 and (50) into equations (47) it results

$$C_{app} = (n-n^2)\alpha_0 + (n^2+n^3)\alpha_1 \quad (51)$$

$$C_{app} = (-2+n-n^2)\alpha_0 + (2-6n+n^2+n^3)\alpha_1 \quad (52)$$

Applying the boundary condition

$\emptyset = 1$ at $n = 0$, $\emptyset = 0$ at $n = \infty$

When $\emptyset = 1$, $n = 0$

$$\emptyset(0)_{app} = (-2+n-n^2)\alpha_0 + (2-0+0-0)\alpha_1 \quad (53)$$

$$1 = -2\alpha_0 + 2\alpha_1$$

$$\frac{1}{2} = \alpha_1 - \alpha_0 \quad (54)$$

Where $\emptyset = 0$, $n = 0$

$$\lim_{n\to\infty} \emptyset(n) = (-2+n-n^2)\alpha_0 + (2-6n+n^2-n^3)\alpha_1 \quad (55)$$

$$\text{Lim} \quad \emptyset(n) = \left[-\frac{2}{n^2}+\frac{n}{n^2}-\frac{n^2}{n^2}\right]\alpha_0 + \left[\frac{2}{n^3}-\frac{6n}{n^3}+\frac{n^2}{n^3}-\frac{n^3}{n^3}\right]\alpha_1 \quad (56)$$

$$= \left[-2\left(\frac{1}{n}\right)^2 + \frac{1}{n} - 1\right]\alpha_0 + \left[2\left(\frac{1}{n}\right)^3 - 6\left(\frac{1}{n}\right)^2 + \frac{1}{n} - 1\right]\alpha_1 \quad (57)$$

$$n \to \infty, \qquad \frac{1}{n} \to 0$$

$$\emptyset(n) = [-2(0)+0-1]\alpha_0 + [2(0)^3 - 6(0) + 0 - 1]\alpha_1 \quad (58)$$

$$\emptyset(n) = -\alpha_0 - \alpha_1 \quad (59)$$

$$0 = -\alpha_0 - \alpha_1 \quad (60)$$

Combining equations (54) and (60)

$$\alpha_0 - \alpha_1 = -\frac{1}{2} \quad (61)$$

$$\alpha_0 - \alpha_1 = 0 \quad (62)$$

$$\alpha_0 = -\frac{1}{4} \quad (63)$$

Put equations (63) into equations (60)

$$\alpha_1 = \frac{1}{4} \quad (64)$$

But $\emptyset(n)_{app} \quad (-2+n-n^2)\alpha_0 + (2-6n+n^2)\alpha_1$ (65)

$$\alpha_0 = -\frac{1}{4}, \qquad \alpha_1 = \frac{1}{4} = -k_r x_2 + 12Scx_3 + 2x_2 b_0 + x_1 b_1 \quad (66)$$

$$\emptyset(n)_{app} = \{-\frac{1}{4} \times (-2+n-n^2)\} + \{(2-6n+n^2) \times \{-k_r x_2 + 12Scx_3 + 2x_2 b_0 + x_1 b_1\} \quad (67)$$

$$\emptyset(n)_{app} = \frac{1}{2} - \frac{n}{4} + \frac{n^2}{4} - 2K_r x_1 + 12Scx_3 + 4x_2 b_0 + 2x_1 b_1 + 6nk_r x_1 - 36Scx_3 n - 12k_r n b_0 - 6x_1 b_1 n - n^2 k_r x_1 + 6Scn^2 x_3 + 2n^2 x_2 b_0 + n^2 x_1 b_1 + n^3 k_r x_1 - 6n^3 Scx_3 - 2n^3 x_2 b_0 - n^3 x_1 b_1 \quad (68)$$

$$\emptyset(n)_{app} = \left[n^3 k_r x_1 - 6n^3 Scx_3 - 2n^3 x_2 b_0 - n^3 x_1 b_1\right] + \left[\frac{n^2}{4} - n^2 k_r x_1\ 6n^2 Scx_3 + 2n^2 x_2 b_0 + n^2 x_1 b_1\right] + \left[-\frac{n}{4} + 6nk_r x_1 - 36Scx_3 n - 12nx_2 b_0 - 6nb_1 x_1\right] + \left[\frac{1}{2} - 2k_r x_1 + 12Scx_3 + 4x_2 b_0 + 2x_1 b_1\right] \quad (69)$$

Consider $x_1 = x_2 = x_3 = 1$, $b_0 = b_1 = 1$.

Then equations (69) becomes

$$\emptyset(n)_{app} = \left[k_r - 6Sc - 3\right]n^3 + \left[-k_r + 6Sc + \frac{13}{4}\right]n^2 + \left[6k_r - 36Sc - \frac{83}{4}\right]n + \left[2k_r + 12Sc + \frac{25}{4}\right] \quad (70)$$

Equation (70) therefore is the approximate solution of the concentration equation.

For the energy equation,

$$\theta(n) = a_0 + a_1 n + a_2 n^2 + a_3 n^3 + a_4 n^4 + a_5 n^5 + a_6 n^6 + a_7 n^7 + a_8 n^8 \quad (71)$$

$$\theta'(n) = a_1 + 2a_2 n + 3a_3 n^2 + 4a_4 n^3 + 5a_5 n^4 + 6a_6 n^5 + 7a_7 n^6 + 8a_8 n^7 \quad (72)$$

$$\theta''(n) = 2a_2 + 6a_3 n + 12a_4 n^2 + 20a_5 n^3 + 30a_6 n^4 + 42a_7 n^5 + 56a_8 n^6 \quad (73)$$

$$f(n) = b_0 + b_1 n + b_2 n^2 + b_3 n^3 + b_4 n^4 + b_5 n^5 + b_6 n^6 + b_7 n^7 + b_8 n^8 \quad (74)$$

$$b_1 + 2b_2 n + 3b_3 n^2 + 4b_4 n^3 + 5b_5 n^4 + 6\,b_6 n^5 + 7b_7 n^6 + 8b_8 n^7 \quad (75)$$

$$2b_2 + 6b_3 n + 12b_4 n^2 + 20b_5 n^3 + 30b_6 n^4 + 42b_7 n^5 + 56b_8 n^6 \quad (76)$$

$$(\text{Pr} - R)\{2a_2 + 6a_3 n + 12a_4 n^2 + 20a_5 n^3 + 30a_6 n^4 + 42a_7 n^5 + 56a_8 n^6\} + Q\{a_0 + a_1 n + a_2 n^2 + a_3 n^3 + a_4 n^4 + a_5 n^5 + a_6 n^6 + a_7 n^7 + a_8 n^8\} + \{b_0 + b_1 n + b_2 n^2 + b_3 n^3 + b_4 n^4 + b_5 n^5 + b_6 n^6 + b_7 n^7 + b_8 n^8\}\{a_1 + 2a_2 n + 3a_3 n^2 + 4a_4 n^3 + 5a_5 n^4 + 6a_6 n^5 + 7a_7 n^6 + 8a_8 n^7\} + \text{Pr}\{2b_2 + 6b_3 n + 12b_4 n^2 + 20b_5 n^3 + 30b_6 n^4 + 42b_7 n^5 + 56b_8 n^6\}\{2b_2 + 6b_3 n + 12b_4 n^2 + 20b_5 n^3 + 30b_6 n^4 + 42b_7 n^5 + 56b_8 n^6\} = 0 \quad (77)$$

Simplifying equations (77) and collecting like terms we have

$$2a_2\,\text{Pr} - 2aR + Qa_1 - a_1 b_0 \quad (78)$$

$$\{24Prb_2 b_3 + 2a_2 b_0 + 2Qa_2 - 6a_2\,\text{Pr} + 6a_2 R\}n \quad (79)$$

$$\left[12a_4\,\text{Pr} - 12a_4 R - 3a_3 Q + 3a_3 b_o + 2a_2 b_2 + b_1 a_1 + 24Prb_2 b_4 + 36Prb_3^2 + 24Prb_4 b_2\right]n^2 \quad (80)$$

$[20a_5\,\mathrm{Pr} - 20a_5R + 4a_4b_0 + 3b_1a_3 + 2a_2b_2 + a_1b_3 + 40Prb_2b_3 + 72Prb_3 + 72Prb_3b_4 + 40Prb_2b_8]n^3$ (81)

$[30a_0\,\mathrm{Pr} - 30a_6R + 5a_5Q + 5a_3b_0 + 4a_4b_1 + 3a_3b_2 + 2a_2b_3 + a_1b_4 + 60Prb_2b_6 + 120Prb_5 + 120Prb_5 + 144Prb_4^2 + 120Prb_3b_5 + 60Prb_2b_6]n^4$ (82)

$[42a_7\,\mathrm{Pr} - 42a_7R + 6Qa_6 + 5a_3b_1 + 4a_4b_2 + 3a_3b_3 + 2a_2b_4 + a_1b_5 + 84Prb_2b_7 + 180Prb_3b_6 + 240Prb_3b_4 + 240Prb_4b_5 + 180b_3b_6 + 84Prb_2b_7]n^5$ (83)

$[\,56Pra_8 - 56a_8R + 7a_7Q + 7a_7b_0 + 6a_6b_1 + 5a_3b_2 + 4a_4b_3 + 3a_3b_4 + 2a_2b_5 + a_1b_6 + 112Prb_2b_8 + 252Prb_3b_7 + 360Prb_4b_6 + 400Prb_8^2 + 360Prb_4b_6 + 252Prb_3b_7 + 112Prb_2b_8]n^6$ (84)

$[\,8a_8Q + 8a_8b_0 + 7a_7b_1 + 6a_6b_2 + 5a_3b_3 + 4a_4b_4 + 3a_3b_5 + 2a_2b_6 + a_1b_7 + 336Prb_3b_8 + 560Prb_4b_4 + 1200Prb_6b_8 + 504Prb_4b_7 + 336Prb_3b_8]n^7$ (85)

$[\,8a_8b_1 + 7a_7b_2 + 6a_6b_3 + 5a_3b_4 + 4a_4b_5 + 3a_3b_6 + 2a_2b_2 + a_1b_8 + 672Prb_5b_8 + 800Prb_5b_7 + 900Prb_0^2 + 840Prb_5b_7 + 672Prb_4b_8\,]n^8$ (86)

$[\,8a_8b_2 + 7a_7b_3 + 6a_6b_4 + 5a_3b_5 + 4a_4b_6 + 3a_3b_7 + 1120Prb_5b_8 + 1260Prb_6b_7 + 1260Prb_6b_7 + 1120Prb_5b_8]n^9$ (87)

$[\,8a_8b_3 + 7a_7b_4 + 6a_5b_5 + 5a_3b_6 + 4a_4b_7 + 3a_3b_8 + 1680Prb_6b_8 + 1764Prb_7^2 + 1680Prb_6]n^{10}$ (88)

$[\,8a_8b_4 + 7a_7b_5 + 6a_6b_6 + 5a_3b_7 + 4a_4b_8 + 2352Prb_7b_8]n^{11}$ (89)

$\left[\,8a_8b_5 + 7a_7b_6 + 6a_6b_7 + 5a_3b_8 + 136Prb_8^2\right]n^{12}$ (90)

$[\,8a_8b_6 + 7a_7b_7 + 6a_6b_8]n^{13}$ (91)

$[\,8a_8b_7 + 7a_7b_8\,]n^{14}$ (92)

$8a_8b_8n^{15}$ (93)

To use collocation weighted residual method. substitute equation (71) – (93) into equation (40) it implies

$\alpha_0 + \alpha_1 n + \alpha_2 n^2 + \alpha_3 n^3 + \alpha_4 n^4 + \alpha_5 n^5 + \alpha_6 n^6 + \alpha_7 n^7 + \ldots\ldots\ldots\ldots + \alpha_{15} n^{15}$ (94)

In Collocation weighted residual method

$$\varepsilon_I = -T_{app} + \sum_{k=0}^{\alpha} \alpha_k \theta_k \quad (95)$$

Were ε_I is the internal domain error, T_{app} is the concentration approximate solution, α_k represents the unknown coefficient. θ_k, is the independent function Which from equation (95), θ_k comes from a complete sequence defined by

$\theta_1 = n(1-n)$ (96)

$\theta_2 = n(1-n)n$ (97)

$\theta_3 = n(1-n)n^2$ (98)

To get an approximate solution the internal domain ε_I must be forced to zero i.e

$$0 = -T_{app} + \sum_{k=0}^{\alpha} \alpha_k \theta_k \quad (99)$$

$$T_{app} = \sum_{k=0}^{\alpha} \alpha_k \theta_k \quad (100)$$

For conveniency using only two collocation points $\alpha_0 + \alpha_1 n$ and let $\theta_k = n(1-n)$

$T_{app} = (\alpha_0 - \alpha_1 n)n(1-n)$ (101)

$T_{app} = \alpha_0[\,n(1-n)] + \alpha_1 n[\,n(1-n)]$ (102)

$T_{app} = \alpha_0 n(1-n) + \alpha_1 n^2(1-n)$ (103)

$T_{app} = \alpha_0 n - \alpha_0 n^2 + \alpha_1 n^2 - \alpha_1 n^3$ (104)

$T_{app} = \alpha_0(n-n^2) + (n^2-n^3)\alpha_1$ (104)

For second order Ordinary differential equation

$$T''_{app}(n) = -2 \quad (105)$$

$T_{app}(n) = n^2 - n^3$

$$T''_{app}(n) = 2 - 6n \quad (106)$$

Add equation (105) and (106) into equation (103) it gives

$T_{app} = (-2 + n - n^2)\alpha_0 + (2 - 6n + n^2 - n^3)\alpha_1$ (107)

Applying the boundary conditions where $\theta = 1$ at $n = 0$, $\theta = 0$ at $n = \alpha$

$1 = (-2 + 0 - 0^2)\alpha_0 + (2 - 6(0) + 0^2 - 0^3)\alpha_1$ (108)

$1 = -2\alpha_0 + 2\alpha_1$ (109)

When $\theta = 1$, $n = \alpha$

$\lim_{n\to\infty} \theta(n) = (-2 + n - n^2)\alpha_0 + (2 - 6n + n^2 - n^3)\alpha_1$ (110)

$\lim_{n\to\infty} \theta(n) = \left[-\frac{2}{n^2} + \frac{n}{n^2} - \frac{n^2}{n^2}\right]\alpha_0 + \left[\frac{2}{n^3} - \frac{6n}{n^3} + \frac{n^2}{n^3} - \frac{n^3}{n^3}\right]\alpha_1$ (111)

$\theta(n) = [\,-2(0)^2 + 0 - 1\,]\alpha_0 + [\,2(0)^3 - 6(0)^2 + 0 - 1\,]\alpha_1$ (112)

$\theta(n) = -\alpha_0 - \alpha_1$ (113)

$\theta(\alpha) = 0$

$-\alpha_0 - \alpha_1 = 0$ (114)

Combining equations (114) and equation (109) it gives

$\frac{1}{2} = \alpha_1 - \alpha_0$ (115)

$0 = -\alpha_1 - \alpha_0$ (116)

Therefore $\alpha_0 = -\frac{1}{4}$ (117)

Put equation (117) into equation (116) implies

$\alpha_1 = \frac{1}{4}$ (118)

Recall that, $\theta_{app}(n) = (-2 + n - n^2)\alpha_0 + (2 - 6n + n^2 - n^3)\alpha_1$

$\alpha_0 = -\frac{1}{4}, \quad \alpha_1 = \frac{1}{4} = 24Prb_2b_3 + 2a_2b_0 + 2Qa_2 - 6a_2\,\mathrm{Pr} + 6a_2R$ (119)

$\theta_{app}(n) = -\frac{1}{4}(-2 + n - n^2) + (2 - 6n + n^2 - n^3)24Prb_2b_3 + 2a_2b_0 + 2Qa_2 - 6a_2\,\mathrm{Pr} + 6a_2R$ (120)

$\theta_{app}(n) = \frac{1}{2} - \frac{n}{4} + \frac{n^2}{4} + 24Prb_2b_3 + 4a_2b_0 + 4Qa_2 - 6a_2\,\mathrm{Pr} + 6a_2R - 144nPrb_2b_3 - 12a_2b_3n - 12nQa_2 + 36na_2\,\mathrm{Pr} - 36na_2R + 24n^2Prb_2b_3 + 2n^2a_2b_0\ 2n^2Qa_2 - 6n^2a_2\,\mathrm{Pr} + 6n^2a_0R - 24n^3Prb_2b_3 - 2n^3a_2b_0 - 6n^3a_2\,\mathrm{Pr} - 6n^3a_2R$ (121)

Collecting like terms

$\theta_{app}(n) = [\ -24Prb_2b_3 - 2a_2b_0 - 2Qa_2 + 6a_2\,\mathrm{Pr} - 6a_3R\ ]n^3 + \left[\ 24Prb_2b_3 + 2a_2b_0 + 2Qa_1 - 6a_2\,\mathrm{Pr} + 6a_0R + \frac{1}{4}\ \right]n^2 + \left[\ -144Prb_2b_3 - 12a_2b_0 - 12Qa_2 + 36Pra_2 - 36Ra_2 - \frac{1}{4}\ \right]n + [\ \frac{1}{2} + 24Prb_2b_3 + 4a_2b_0 + 4Qa_2 - 6Pra_2 + 6Ra_2\ ]$ (122)

Considering $a_1 = a_2 = a_3 = 1$

$b_0 = o, \quad b_1 = b_2 = 1$

$\theta_{app}(n) = [\ -18Pr - 2Q - 6R\ ]n^3 + [\ 18Pr + 2Q + 0.25\ ]n^2 + [\ -108Pr - 12Q - 36R - 0.25\ ]n + [\ 18Pr + 4Q + 6R + 0.5\ ]$ (123)

From equation (16) which is the angular momentum equation

$h(n) = y_0 + y_1n + y_2n^2 + y_3n^3 + y_4n^4 + y_5n^5 + y_6n^6 + y_7n^7 + y_8n^8$ (124)

$h'(n) = y_1 + 2y_2n + 3y_3n^2 + 4y_4n^3 + 5y_5n^4 + 6y_6n^5 + 7y_7n^6 + 8y_8n^7$ (125)

$h''(n) = 2y_2 + 6y_3n + 12y_4n^2 + 20y_5n^3 + 30y_6n^4 + 42y_7n^5 + 56y_8n^6$ (126)

$f(n) = b_0 + b_1n + b_2n^2 + b_3n^3 + b_4n^4 + b_5n^5 + b_6n^6 + b_7n^7 + b_8n^8$ (127)

$f'(n) = b_1 + 2b_2n + 3b_3n^2 + 4b_4n^3 + 5b_5n^4 + 6b_6n^5 + 7b_7n^6 + 8b_8n^7$ (128)

$f''(n) = 2b_2 + 6b_3n + 12b_4n^2 + 20b_5n^3 + 30b_6n^4 + 42b_7n^5 + 56b_8n^6$ (129)

Put equations (124) – (129) into equations (14)

$\lambda\,(2y_2 + 6y_3n + 12y_4n^2 + 20y_5n^3 + 30y_6n^4 + 42y_7n^5 + 56y_8n^6) + (y_1 + 2y_2n + 3y_3n^2 + 4y_4n^3 + 5y_5n^4 + 6y_6n^5 + 7y_7n^6 + 8y_8n^7)*$
$(b_0 + b_1n + b_2n^2 + b_3n^3 + b_4n^4 + b_5n^5 + b_6n^6 + b_7n^7 + b_8n^8\ ) - K(2b_2 + 6b_3n + 12b_4n^2 + 20b_5n^3 + 30b_6n^4 + 42b_7n^5 + 56b_8n^6) - 2K(y_0 + y_1n + y_2n^2 + y_3n^3 + y_4n^4 + y_5n^5 + y_6n^6 + y_7n^7 + y_8n^8) - (b_1 + 2b_2n + 3b_3n^2 + 4b_4n^3 + 5b_5n^4 + 6b_6n^5 + 7b_7n^6 + 8b_8n^7\ ) * (\ y_0 + y_1n + y_2n^2 + y_3n^3 + y_4n^4 + y_5n^5 + y_6n^6 + y_7n^7 + y_8n^8)$ (130)

Simplifying equation (130) and collecting like terms.

$2\lambda y_2 + y_1b_0 - 2Kb_2 - 2Ky_0 - b_1y_0$ (131)

$[\ 6\lambda y_3 - y_1b_1 + 2y_2b_0 - 6Kb_3 - 2Ky_1 - b_1y_1 - 2b_2y_0]n$ (132)

$[\ 12\lambda y_4 + y_1b_2 + 2y_2b_1 + 3b_3y_0 - 12Kb_4 - 2Ky_2 - b_1y_2 - 2b_2y_1 - 3b_3y_0]n^2$ (133)

$[\ 20\lambda y_5 + y_1b_3 + 2y_2b_2 + 3b_3y_1 + 4y_4b_0 - 20Kb_5 - b_1y_3 - 2b_2y_2 - 3y_1b_3 - 4b_4y_1\ ]n^3$ (134)

$[\ 30\lambda y_6 + y_1b_4 + 2y_2b_3 + 3b_2y_3 + 4y_4b_1 + 5b_0y_5 - 30Kb_6 - 2Ky_4 - b_1y_4 - 2y_3b_2 - 3y_2b_3 - 4b_4y_1 - 5y_5b_0\ ]n^4$ (135)

$[\ 42\lambda y_7 + y_1b_5 + 2y_2b_4 + 3y_3b_3 + 4y_4b_2 + 5y_5b_1 - 2Ky_5 - 42Kb_7 - b_1y_5 - 2b_2y_4 - 3b_3y_3 - 4b_4y_2 - 6b_5y_6\ ]n^5$ (136)

$[\ 56\lambda y_3 + y_1b_0 + 2y_2b_5 + 3b_4y_3 + 5y_5b_2 + 6y_6b_1 + 7y_7b_0 - 56Kb_8 - 2Ky_6 - b_1y_6 - 2y_5b_2 - 3y_3b_4 - 4y_4y_4 - 5b_5y_2 - 6b_8y_1\ ]n^6$ (137)

$[\ y_1b_7 + 2y_2b_6 + 3y_3b_5 + 4b_4y_6 + 5y_5b_3 + 7y_7b_1 + 8y_8b_0 - 2Kb_7 - b_1y_7 - 2b_2y_7 - 3y_6b_3 - 4b_4y_4 - 5b_5y_4 - 6b_8y_2 - 7b_7 - 8b_8y_0]n^7$ (138)

$[\ y_1b_8 + 2y_2b_7 + 3b_6y_5 + 4y_4b_5 + 5b_4y_5 + 7y_7b_2 + 8y_8b_1 - 2Ky_8 - b_1y_8 - 2b_2y_7 - 3y_6b_3 - 4b_4y_5 - 5b_5y_4 - 6b_8y_3 - 7y_2b_7\ ]n^8$ (139)

$[\ 2y_2b_8 + 4y_4b_7 + 6b_4y_6 + 7y_7b_3 + 8y_8b_2 - 2b_2y_8 - 3b_3y_7 - 4b_4y_6 - 5b_5y_5 - 6b_8y_4 - 7b_7y_3 - 8y_2b_8\ ]n^9$ (140)

$[\ 4y_4b_7 + 5b_6y_5 + 7y_7b_4 + 8y_5b_3 - 3b_3y_8 - 4b_4y_7 - 5b_5y_4 - 6b_5y_6 - 7b_7y_4 - 8y_8b_8]n^{10}$ (141)

$[\ 4b_8y_4 + 7y_7b_5 + 8y_5b_3 - 4b_4y_8 - 5y_7b_5 - 6b_8y_6 - 7b_7y_5 - 8b_8y_4\ ]n^{11}$ (142)

$[\ 7y_7b_6 + 8y_8b_4 - 5y_8b_5 - 6b_8y_7 - 7y_6b_7 - 8y_5b_8\ ]n^{12}$ (143)

$[\ 7y_7b_7 + 8y_8b_5 - 6y_8b_6 - 7y_7b_7 - 8b_8y_8\ ]n^{13}$ (144)

$[\ 7y_7b_8 + 8y_8b_6 - 7b_7y_8 - 8b_8y_7\ ]n^{14}$ (145)

To use collocation weighted residual method substitute equation (131) – (145) into equation (40) it implies

$\alpha_0 + \alpha_1n + \alpha_2n^2 + \alpha_3n^3 + \alpha_4n^4 + \alpha_5n^5 + \alpha_6n^6 + \alpha_7n^7 + \cdots \ldots \ldots \ldots \ldots + \alpha_{14}n^{14}$

In Collocation weighted residual method

$$\varepsilon_I = -h_{app} + \sum_{k=0}^{\infty}\alpha_k N_k \quad (146)$$

Were ε_I is the internal domain error, h_{app} is the concentration approximate solution, α_k represents the unknown coefficient. N_k, is the independent function Where N_k comes from a complete sequence defined by.

$N_1 = n(1-n)$ (147)

$N_2 = n(1-n)n$ (148)

$N_3 = n(1-n)n^2$ (149)

To get an approximate solution, the interval domain error ε_I is equal to zero

$$h_{app} = \sum_{k=0}^{\infty} \alpha_k N_k \quad (150)$$

Using only two collocation points $\alpha_0 + \alpha_1 n$

$N_k = n(1-n)$ (151)

$h_{app} = (\alpha_0 + \alpha_1 n)n(1-n)$ (152)

$h_{app} = \alpha_0[n(1-n)] + \alpha_1 n[n(1-n)]$ (153)

$h_{app} = (n-n^2)\alpha_0 + (n^2-n^3)\alpha_1$ (154)

Equation (154) can be written in second order differential equation

$h_{app}(n) = (-2+n-n^2)\alpha_0 + (2-6n+n^2-n^3)\alpha_1$ (156)

Applying boundary conditions in equation (156)

at $n = 0$ (157)

$h \to 0 \qquad n \to \infty$

Equation (156) becomes

$-n\ f'' = (-2+0-0^2)\alpha_0 + (2-6(0)+(0)^2+(0)^3)\alpha_1$ (158)

$= -2\alpha_0 + 2\alpha_1$ (159)

Where $h \to 0, \quad n \to \infty$

Lim $h(n) = (-2+n-n^2)\alpha_0 + (2-6n+n^2-n^3)\alpha_1$ (160)

$\lim h(n) = \left[-\frac{2}{n^2} + \frac{n}{n^2} - \frac{n^2}{n^2} \right]\alpha_0 + \left[\frac{2}{n^3} - \frac{6n}{n^3} + \frac{n^2}{n^3} - \frac{n^3}{n^3} \right]\alpha_1$ (161)

$= \left[-2\left(\frac{1}{2}\right)^2 + \frac{1}{n} - 1 \right]\alpha_0 + \left[2\left(\frac{1}{n}\right)^3 - 6\left(\frac{1}{n}\right)^2 + \frac{1}{n} - 1 \right]\alpha_1$ (162)

As $n \to \alpha, \quad \frac{1}{n} \to 0$

$h(n) = \left[-2(0)^2 + 0 - 1 \right]\alpha_0 + \left[2(0)^3 - 6(0)^2 + 0 - 1 \right]\alpha_1$ (163)

$h(\infty) = -\alpha_0 - \alpha_1$

$h(\infty) = 0$

$0 = -\alpha_0 - \alpha_1$ (164)

Combining equation (163) and equation (164)

$0 = -\alpha_0 - \alpha_1$

But

$h_{app}(n) = (-2+n-n^2)\alpha_0 + (2-6n+n^2-n^3)\alpha_1$

$-n\ f'' = \alpha_0, \quad n\ f'' = \alpha_1$

$\alpha_0 = -n\ f'' = 2\lambda y_2 + y_1 b_0 - 2Kb_2 - 2Ky_0 - b_1 y_0$

, $\alpha_1 = n\ f''$

$h_{app}(n) = (-2+n-n^2) \times (2\lambda y_2 + y_1 b_0 - 2Kb_2 - 2Ky_0 - b_1 y_0) + (2-6n+n^2-n^3)\ nf''$ (165)

$h_{app}(n) = -4\lambda y_4 - 2y_1 b_0 + 4Kb_2 + 2b_1 y_0 + 2n\lambda y_2 + n y_1 b_0 - 2nKb_2 - 2nKy_0 - nb_1 y_0 - 2n^2\lambda y_2 - n^2 y_1 b_0 + 2n^2 K y_0 + nb_1 y_0 + 2\ f'' - 6n^2\ f'' + n^3\ f'' - n^4$ (166)

Recall that $2b_2 + 6b_3 n + 12b_4 n^2 + 20b_5 n^3 + 30b_6 n^4 + 42b_7 n^5 + 56b_8 n^6$

Hence equation (166) becomes

$h_{app}(n) = -4\lambda y_2 - 2y_1 b_0 + 4Kb_2 + 4Ky_0 + 2n\lambda y_2 + n y_1 b_0 - 2nKb_2 - 2nKy_0 - nb_1 y_0 - 2n^2 y_2 - n^2 y_1 b_0 + 2n^2 Kb_2 + 2n^2 Ky_0 + nb_1 y_0 + 4nb_2 + 12n^2 b_3 + 24n^3 b_4 + 40n^4 b_5 + 60n^5 b_6 + 82n^6 b_7 + 112n^7 b_8 - 12n^2 b_2 - 36n^3 b_3 - 72n^4 b_4 - 120n^5 b_5 - 180n^6 b_6 - 252n^7 b_7 - 366n^8 b_8 + 2b_2 n^3 + 6b_3 n^4 + 12b_4 n^5 + 20b_5 n^6 + 30b_6 n^7 + 42b_7 n^8 - 56b_8 n^9 - 2b_2 n^4 - 6b_3 n^5 - 12b_4 n^6 - 20b_5 n^7 - 30b_6 n^8 - 42b_7 n^9 - 56b_8 n^{10}$ (167)

$h_{app}(n) = [-56b_8]n^{10} - [56b_8 + 42b_7]n^9 + [-366b_8 + 42b_7 - 30b_6]n^8 + [112b_8 - 252b_7 + 30b_6 - 20b_5]n^7 + [82b_7 - 180b_6 + 20b_5 - 12b_4]n^6 + [60b_6 + 12b_4 - 120b_5 - 6b_3]n^5 + [40b_5 - 72b_4 + 6b_3 - 2b_2]n^4 + [24b_4 - 36b_3 + 2b_2]n^3 + [-2y_2 - y_1 b_0 + 2Kb_2 + 2Ky_0 + 12b_3]n^2 + [2\lambda y_2 + y_1 b_0 - 2Kb_2 - 2Ky_0]n + [-4\lambda y_2 - 2y_1 b_0 + 4Kb_2 + 4Ky_0 + 2b_1 y_0]$ (168)

Considering

$b_8 = b_7 = b_6 = b_5 = b_4 = b_3 = b_2 = b_1 = b_0 = 1$

$y_2 = y_1 = y_0 = 1$ in equations (168), it results

$h_{app}(n) = -56n^{10} - 98n^9 - 352n^8 - 130n^7 + 90n^6 - 44n^5 - 28n^4 - 10n^3 + [4K - 15]n^2 + [2\lambda - 4K]n + [-4\lambda + 8K]$ (169)

Recall

$\theta(n) = a_0 + a_1 n + a_2 n^2 + a_3 n^3 + a_4 n^4 + a_5 n^5 + a_6 n^6 + a_7 n^7 + a_8 n^8$

$\emptyset(n) = x_0 + x_1 n + x_2 n^2 + x_3 n^3 + x_4 n^4 + x_5 n^5 + x_6 n^6 + x_7 n^7 + x_8 n^8$

$f(n) = b_0 + b_1 n + b_2 n^2 + b_3 n^3 + b_4 n^4 + b_5 n^5 + b_6 n^6 + b_7 n^7 + b_8 n^8$

$f'(n) = b_1 + 2b_2 n + 3b_3 n^2 + 4b_4 n^3 + 5b_5 n^4 + 6b_6 n^5 + 7b_7 n^6 + 8b_8 n^7$

$f''(n) = 2b_2 + 6b_3 n + 12b_4 n^2 + 20b_5 n^3 + 30b_6 n^4 + 42b_7 n^5 + 56b_8 n^6$

$f'''(n) = 6b_3 + 24b_4 n + 60b_5 n^2 + 120b_6 n^3 + 210b_7 n^4 + 392b_8 n^5$

$h(n) = y_0 + y_1 n + y_2 n^2 + y_3 n^3 + y_4 n^4 + y_5 n^5 + y_6 n^6 + y_7 n^7 + y_8 n^8$

$h'(n) = y_1 + 2y_2 n + 3y_3 n^2 + 4y_4 n^3 + 5y_5 n^4 + 6y_6 n^5 + 7y_7 n^6 + 8y_8 n^7$

Put the above equations into equation (13) results

$K(6b_3 + 24b_4 n + 60b_5 n^2 + 120b_6 n^3 + 210b_7 n^4 + 392b_8 n^5) + K(y_1 + 2y_2 n + 3y_3 n^2 + 4y_4 n^3 + 5y_5 n^4 + 6y_6 n^5 + 7y_7 n^6 + 8y_8 n^7) - (b_1 + 2b_2 n + 3b_3 n^2 + 4b_4 n^3 + 5b_5 n^4 + 6b_6 n^5 + 7b_7 n^6 + 8b_8 n^7) \times (b_1 + 2b_2 n + 3b_3 n^2 + 4b_4 n^3 + 5b_5 n^4 + 6b_6 n^5 + 7b_7 n^6 + 8b_8 n^7) - \sigma(b_1 + 2b_2 n + 3b_3 n^2 + 4b_4 n^3 + 5b_5 n^4 + 6b_6 n^5 + 7b_7 n^6 + 8b_8 n^7) - M(b_0 + b_1 n + b_2 n^2 + b_3 n^3 + b_4 n^4 + b_5 n^5 + b_6 n^6 + b_7 n^7 + b_8 n^8) + (b_1 + 2b_2 n + 3b_3 n^2 + 4b_4 n^3 + 5b_5 n^4 + 6b_6 n^5 + 7b_7 n^6 + 8b_8 n^7) \times (b_0 + b_1 n + b_2 n^2 + b_3 n^3 + b_4 n^4 + b_5 n^5 + b_6 n^6 + b_7 n^7 + b_8 n^8) + G_r(a_0 + a_1 n + a_2 n^2 + a_3 n^3 + a_4 n^4 + a_5 n^5 + a_6 n^6 + a_7 n^7 + a_8 n^8) + G_m(x_0 + x_1 n + x_2 n^2 + x_3 n^3 + x_4 n^4 + x_5 n^5 + x_6 n^6 + x_7 n^7 + x_8 n^8)$ (170)

Collecting like terms

$6Kb_3 + Ky_1 - b_1^2 - \sigma b_1 - Mb_0 - b_0 b_1 - G_r a_0 + G_m x_0$ (171)

$[24Kb_4 + 2Ky_2 - 2b_1 b_2 - 2\sigma b_2 - Mb_1 - b_1^2 + G_r a_1 + G_m x_1]n$ (172)

$[60Kb_6 + 3Ky_3 - 3b_1 b_3 - 4b_2^2 - 3b_1 b_3 - 3\sigma b_3 - Mb_2 - 3b_0 b_3 - 2b_1 b_2 - b_1 b_2 + G_r x_2 + G_m x_3]n^2$ (173)

$[120Kb_6 + 4Ky_4 - 4b_1 b_4 + 12b_2 b_3 - 4b_1 b_4 - 4\sigma b_4 - 3b_2 b_3 - 2b_2^2 + G_r a_3 + G_m x_3]n^3$ (174)

$[210Kb_7 + 5Ky_5 - 5b_1 b_5 - 9b_3^2 - 8b_2 b_4 - 5b_1 b_2 - 3b_2 b_3 + G_r a_4 + G_m x_4]n^4$ (175)

$[392Kb_8 - 6b_1 b_6 - 10b_2 b_5 - 12b_3 b_4 - 12b_3 - 10b_2 b_5 - 6b_1 b_6 - 6\sigma b_6 - Mb_5 - 6b_0 b_6 - 5b_2 b_5 - 3b_4 b_3 - 2b_5 b_2 - b_1 b_6 + G_r a_6 + G_m x_5]n^5$ (176)

$[7Ky_7 - 7b_1 b_7 - 12b_2 b_6 - 15b_5 b_3 - 16b_4 - 15b_3 b_5 - 7b_1 b_7 - 6\sigma b_6 - Mb_5 - 6b_0 b_6 - 5b_2 b_5 - 3b_4 b_3 - 2b_5 b_2 - b_1 b_6 + G_r a_6 + G_m x_0]n^6$ (177)

$[8Ky_8 - 8b_1 b_8 - 14b_2 b_7 - 16b_2 - 21b_3 b_7 - 40b_4 b_5 - 18b_3 b_6 - 14b_2 - 8b_1 b_8 - 8\sigma b_8 - Mb_7 - 8b_0 b_8 - 7b_1 b_7 - 6b_2 b_6 - 4b_4^2 - 3b_5 b_3 - 2b_6 b_2 - b_1 b_7]n^7$ (178)

$[-21b_3 b_7 - 48b_4 b_6 - 25b_5^2 - 21b_3 b_7 - 16b_2 b_8 - Mb_8 - 8b_1 b_8 - 7b_2 b_7 - 5b_4 b_5 - 4b_4 b_5 - 3b_6 b_5 - 2b_2 b_7 - b_1 b_8 + G_r a_8 + G_m x_6]n^8$ (179)

$[-24b_8 b_3 - 28b_4 b_7 - 30b_6^2 b_5 - 28b_4 - 24b_3 b_8 - 8b_8 b_2 - 6b_4 b_6 - 5b_5^2 - 4b_4 b_6 - 3b_3 b_7 - 2b_2 b_8]n^9$ (180)

$[-32b_4 b_8 - 35b_5 b_7 - 36b_6^2 - 35b_5 b_7 - 32b_4 b_8 - 7b_5 b_7 - 6b_5 b_6 - 5b_6 b_5 - 4b_4 b_7 - 3b_8 b_6]n^{10}$ (181)

$[-40b_5 b_8 - 42b_6 b_7 - 42b_6 b_7 - 40b_8 b_5 - 8b_4 b_8 - 7b_5 b_7 - 6b_8^2 - 5b_7 b_5 - 4b_8 b_4]n^{11}$ (182)

$[-48b_6 b_8 - 49b_7^2 - 48b_6 b_8 - 8b_8 b_3 - 7b_6 b_7 - 6b_6 b_7 - 5b_8 b_5]n^{12}$ (183)

$[-56b_7 b_8 - 56b_7 b_8 - 8b_6 b_8 - 7b_7^2 - 6b_6 b_8]n^{13}$ (184)

$[-64b_5^2 - 8b_7 b_8 - 7b_8 b_7]n^{14}$ (185)

$-8b_5^2 n^{15}$ (186)

To use collocation weighted residual method. substitute equation (171) – (186) into equation (40) it implies

$\alpha_0 + \alpha_1 n + \alpha_2 n^2 + \alpha_3 n^3 + \alpha_4 n^4 + \alpha_5 n^5 + \alpha_6 n^6 + \alpha_7 n^7 + \cdots \ldots \ldots \ldots \ldots + \alpha_{15} n^{15} = 0$

In Collocation weighted residual method

$$\varepsilon_I = -F_{app} + \sum_{k=0}^{\infty} \alpha_1 U_k$$

Were ε_I is the internal domain error, F_{app} is the concentration approximate solution, α_k represents the unknown coefficient. U_k, is the independent function

Where U_k comes from a complete sequence defined by

$U_1 = n(1-n)$

$U_2 = n(1-n)n$

$U_3 = n(1-n)n^2$

To get the approximate solution the internal domain error ε_I is equal to zero

$$-F_{app} = \sum_{k=0}^{\infty} \alpha_1 U_k$$

Using only two collocation point i.e let us choose $\alpha_0 + \alpha_1 n$ and let $U_k = n(1-n)$

$F_{app} = (n - n^2)\alpha_0 + (n^2 - n^3)\alpha_1$ (187)

For 2nd order differential equation

$f'_{app} = 1 - 2n, \quad f''_{app} = -2$ (188)

$F_{app} = n^2 - n^3$

$f'_{app} = 2n - 3n^2 \qquad f''_{app} = 2 - 6n$ (189)

Add equation (188) and (189) into equation (187)

$F_{app} = (-2 + n - n^2)\alpha_0 + (2 - 6n + n^2 - n^3)\alpha_1$ (190)

Applying the boundary conditions

$f' = 1, \quad f = 0 \quad at\ n = 0, \quad f' \to 0, \quad n \to \infty$

Using the boundary condition F(n) = 0 where n = 0

$F_{app} = (-2 + 0 - 0^2)\alpha_0 + (2 - 6(0) + 0^2 - 0^3)\alpha_1$

$F_{app} = -2\alpha_0 + 2\alpha_1$ (191)

$F(n) = f(0) = 0$ (192)

Put equation (192) into equation (191)
0 = -2$\alpha_0 + 2\alpha_1$
$\alpha_0 = \alpha_1$ (193)
Applying the boundary condition $f'(n) = 1 \quad at\ n = 0$
$\alpha_0 - 6\alpha_1 = 1$(194)
Put equation (277) into equation (278)
$\alpha_1 - 6\alpha_1 = 1, \quad -5\alpha_1 = 1, \quad \alpha_0 = -\frac{1}{5},$
Therefore $\alpha_1 = -\frac{1}{5}$
Using the boundary condition $f' \to 0 \quad at\ n \to \infty$
$If\ n \to \infty, \quad \frac{1}{n} \to 0$
Lim
$n \to \infty$
$0 = (0-2)\alpha_0 + (-6(0)^2 + 2(0) - 3)\alpha_1$
$0 = -2\alpha_0 - 3\alpha_1$ (195)
$2\alpha_0 = -3\alpha_1$ (196)
Combing equation (195) and (196) it gives
$\alpha_1 = -\frac{2}{15}$ (197)
Put equation (195) into equation (197)
$\alpha_0 = \frac{1}{5}$
Therefore $\alpha_1 = -\frac{2}{15}, \quad \alpha_0 = \frac{1}{5}$
But $F_{app} = (-2 + n - n^2)\alpha_0 + (2 - 6n + n^2 - n^3)\alpha_1$
$\alpha_0 = \frac{1}{5}, \ \alpha_1 = -\frac{2}{15} = 24Kb_4 + 2Ky_2 - 2b_1b_2 - 2\sigma b_2 - Mb_1 - b_1^2 + G_r a_1 + G_m x_1$
$F_{app} = \frac{1}{5} \times (-2 + n - n^2) + (2 - 6n + n^2 - n^3) \times \{24Kb_4 + 2Ky_2 - 2b_1b_2 - 2\sigma b_2 - Mb_1 - b_1^2 + G_r a_1 + G_m x_1\}$ (198)
$F_{app} = (24Kb_4 - 2Ky_2 + 2b_1b_2 + 2b_2\sigma + Mb_1 + b_1^2 - G_r a_1 - G_m x_0)n^3 + \left(24kb_4 + 2Ky_2 - 2b_1b_2 - Mb_1 - 2b_2\sigma - b_1^2 + G_r a_1 + G_m a_1 - \frac{1}{5}\right)n^2 + \left(-144K - 26Ky_2 + 12b_1b_2 + 12b_2\sigma + 6Mb_1 + 6b_1^2 - 6G_r a_1 - 6G_m x_0 + \frac{1}{5}\right)n + (-\frac{2}{5} + 48Kb_4 + 4Ky_2 - 4b_1b_2 - 4b_2\sigma - 2Mb_1 - 2b_1^2 + 2G_r a_1 + 2G_m x_0)$ (199)
Let $b_1 = b_2 = b_3 = 1, \quad a_1 = x_0 = 1, \quad y_1 = y_2 = 1$
$F_{app}(n) = (-26K + 2\sigma + M - G_r - G_m + 3)n^3 + \left(26K - 2\sigma - M + G_r + G_m - \frac{16}{5}\right)n^2 + \left(-160k + 12\sigma + 6M - G_r - G_m + \frac{91}{5}\right)n + (52k - 4\sigma - 2M + 2G_r + 2G_m - \frac{32}{5})$ (200)
The skin friction, Nusselt number and the Sherwood number are important physical parameters for this type of flow. The skin friction at the plate, which is the non-dimensional form is given by

$\tau = \left(\frac{\partial u}{\partial n}\right) = 3(-26K + 2\sigma + M - G_r - G_m + 3)n^2 + 2\left(26K - 2\sigma - M + G_r + G_m - \frac{16}{5}\right)n + \left(-160k + 12\sigma + 6M - G_r - G_m + \frac{91}{5}\right)$ (201)
The rate of mass transfer coefficient, which in the from dimensional form in term of the Sherwood number, is given by
$S_h = -\left(\frac{\partial \emptyset}{\partial n}\right) = -3\left[k_r - 6Sc - 3\right]n^2 + 2\left[-k_r + 6Sc + \frac{13}{4}\right]n + \left[6k_r - 36Sc - \frac{83}{4}\right]$ (202)
The rate of heat transfer coefficient, which in the non-dimensional form in terms of the Nusselt number, is given by
$N_u = -\left(\frac{\partial \theta}{\partial n}\right) = -3\left[-24Pr - 2Q + 6\,\mathrm{Pr} - 6R\right]n^2 + 2\left[24Pr + 2Q - 6\,\mathrm{Pr} + 6R + \frac{1}{4}\right]n + \left[-144Pr - 12Q + 36Pr - 36R - \frac{1}{4}\right]$(203)

STABILITY OF THE SYSTEM

Following the procedure of linear stability analysis adopted by Rafiki et al. (2013) let the disturbed micropolar fluid flow be a steady basic flow and a small-time dependent disturbance. We perturb the system and perform the standard normal mode analysis of the form.
$F = f + f_e \qquad \theta_0 = \theta + \theta_e \qquad H = f + f_e \qquad \emptyset_0 = \emptyset + \emptyset_e$
We ascribe to all quantities describing the perturbation a dependence on n and t in the form
$e^{-i(a^*n - \varphi^* t)}$
Where φ is a time constant and a*n is the resultant wave number of the disturbance
We write
$(\mathrm{F}, H, \theta_0, \emptyset_0) = \left[F, (n), , H(n)\ \theta_0(n)\emptyset_0(n)\right] expi(a^*n - \varphi^* t)$ (204)
$F = F(N)e^{-i(a^*n - \varphi^* t)}$ (205)
$\frac{dF}{dn} = -ia^*F(n)e^{-i(a^*n - \varphi^* t)} = \left(\frac{d}{dn} + ia^*\right)F$ (206)
$H = H(N)e^{-i(a^*n - \varphi^* t)}$ (207)
$\frac{dH}{dn} = -ia^*H(n)e^{-i(a^*n - \varphi^* t)} = \left(\frac{d}{dn} + ia^*\right)H$ (208)
$\theta = \theta(N)e^{-i(a^*n - \varphi^* t)}$ (209)
$\frac{d\theta}{dn} = -ia^*\theta(n)e^{-i(a^*n - \varphi^* t)} = \left(\frac{d}{dn} + ia^*\right)\theta$ (210)
$\emptyset = \emptyset(N)e^{-i(a^*n - \varphi^* t)}$ (211)
$\frac{d\emptyset}{dn} = -ia^*\emptyset(n)e^{-i(a^*n - \varphi^* t)} = \left(\frac{d}{dn} + ia^*\right)\emptyset$ (212)
Substitute equations (205) – (212) into equations (187) to (190) and make $\frac{d}{dn}$ = D

$k(D^3 - ia^{*3})F + k(D + ia^*)H - \left((D + ia^*)F\right)^2 - \sigma_\infty(D + ia^*)F - F(n)(D + ia^*)F - M(D + ia^*) + G_r\theta(n) + G_m\emptyset(n) = 0$ (213)

$\lambda(D^2 - a^{*2})H + F(D + ia^*)H - K(D^2 - a^{*2})F - 2Hk - FH(D + ia^*) = 0$ (214)

$(Pr - R)\left(D^2 - a^{*2}\right)\theta + F(D + ia^*)\theta + Q\theta + Pr\left[(D^2 - a^{*2})F\right]^2 = 0$ (215)

$\mathrm{Sc}(D^2 - a^{*2})\emptyset + F(D + ia^*)\emptyset - Kr\emptyset = 0$ (216)

From equation (213)

$\frac{1}{G_r}\ [k\left(D^3 - ia^{*3}\right)F + k(D + ia^*)H - \left((D + ia^*)F\right)^2 - \sigma_\infty(D + ia^*)F - F(D + ia^*)F - M(D + ia^*) + G_m\emptyset\ ] = -\theta$ (217)

From equation (214)

$[\lambda(D^2 - a^{*2}) - 2k]H = K(D^2 - a^{*2})F$ (218)

From equation (215)

$\left[(Pr - R)\left(D^2 - a^{*2}\right) + F(D + ia^*) + Q\right]\theta = -Pr\left[(D^2 - a^{*2})F\right]^2$ (218)

From equation (216)

$[\mathrm{Sc}(D^2 - ia^{*2}) + F(D + ia^*) - Kr]\emptyset = 0$ (219)

Substitute equation (217) into equation (218)

$\left[(Pr - R)\left(D^2 - a^{*2}\right) + F(D + ia^*) + Q\right]$

$[k\left(D^3 - ia^{*3}\right)F + k(D + ia^*)H - \left((D + ia^*)F\right)^2 - \sigma_\infty(D + ia^*)F - F(D + ia^*)F - M(D + ia^*) + G_m\emptyset\ ] = G_rPr\left[(D^2 - a^{*2})F\right]^2$ (220)

let

$\beta_1 = \left[(Pr - R)\left(D^2 - ia^{*2}\right) + F(D + ia^*) + Q\right]$

$\beta_2 = [\ k\left(D^3 - ia^{*3}\right)F + k(D + ia^*)H - \left((D + ia^*)F\right)^2 - \sigma_\infty(D + ia^*)F - F(D + ia^*)F - M(D + ia^*)\ ]$

Therefore equation (220) becomes

$\beta_1\ G_m\emptyset = G_rPr\left[(D^2 - a^{*2})F\right]^2 - \beta_1\beta_2$ (221)

From equation (218)

$[\ (\frac{\lambda}{k}(D^2 - a^{*2}) - 2k)H\ ] = (D^2 - ia^{*2})F$ (222)

Put equation (222) into equation (218)

$\beta_1\ G_m\emptyset = G_rPr\left[\ (\frac{\lambda}{k}(D^2 - a^{*2}) - 2)H\right]^2 - \beta_1\beta_2$

$\emptyset = [\ \frac{1}{\beta_1\ G_m}\ [\ G_rPr\left[\ (\frac{\lambda}{k}(D^2 - a^{*2}) - 2)H\right]^2 - \beta_1\beta_2\ \ ]$ (223)

Substitute equation (223) into equation (218)

$[\mathrm{Sc}(D^2 - ia^{*2}) + F(D + ia^*) - Kr][\ \frac{1}{\beta_1\ G_m}\ [\ G_rPr\left[\ \left(\frac{\lambda}{k}\left(D^2 - a^{*2}\right) - 2\right)H\right]^2 - \beta_1\beta_2\ ] = 0$ (224)

Let $\beta_3 = [\mathrm{Sc}(D^2 - ia^{*2}) + F(D + ia^*) - Kr]$

$\beta_3[\ \frac{1}{\beta_1\ G_m}\ [\ G_rPr\left[\ \left(\frac{\lambda}{k}\left(D^2 - ia^{*2}\right) - 2\right)H\right]^2 - \beta_1\beta_2\ ] = 0$ (225)

According to Bird et al. (2003), the analysis of free convection heat transfer from a vertical plate, shows that $G_r\ \mathrm{Pr} = Ra,$ where Ra is the Rayleigh's number therefore rewrite equation (225) as

$\beta_3\left[Ra\ \left(\frac{\lambda}{k}\left(D^2 - ia^{*2}\right) - 2\right)H\right]^2 - \beta_1\beta_2\beta_3 = 0$ (226)

$\beta_1\beta_2\beta_3 = \beta_3\left[Ra\ \left(\frac{\lambda}{k}\left(D^2 - a^{*2}\right) - 2\right)H\right]^2$ (227)

[(Pr-R)(D^2 〖- a〗 ^(*^2))+F(D+ia^*)+Q] [(D^3 〖- ia〗 ^(*^3))F+k(D+ia^*)H-((D+ia^*)F)^2-σ_∞ (D+ia^*)F-F(D+ia^*)F-M(D+ia^*)] [Sc(D^2-a^(*^2)) + F(D+ia^*)-Kr] = [Sc(D^2-a^(*^2)) + F(D+ia^*)-Kr][Ra (λ/k (D^2-a^(*^2))-2)H]^2 (228)

Following the method adopted by Hocking (1968), the proper solution for F(n) and H(n) appropriate for the lowest mode is

$H(n) = Asin\pi n$ (229)

$F(n) = Asin\pi n$

Where A is a constant

Substitute equation (229) into equation (228) and simplify, the conclusion will be that all the even derivatives of H(n) and F(n) must vanish for n = 0 or n = 1 and the result will be

$[Sc(a^{*2} - \pi^2) + (ia^* + \pi) - Kr]\ \left[(Pr - R)(a^{*2} - \pi^2) + (ia^* + \pi) + Q\right]\ [\ (-ia^{*3} - \pi^3) + k(ia^* + \pi) - \left((ia^* + \pi)\right)^2 - \sigma_\infty(ia^* + \pi) - (ia^* + \pi) - M(ia^* + \pi)\ ] = [Sc(a^{*2} - \pi^2) + (ia^* + \pi) - Kr]\ \left[Ra\ \left(\frac{\lambda}{k}(a^{*2} - \pi^2) - 2\right)\right]^2$ (330)

RESULTS AND DISCUSSION

The numerical solutions of the transformed governing equations were obtained using the collocation weighted residual method. The computed results are presented to examine the effects of the Prandtl number, thermal radiation parameter, heat-function parameter and micropolar parameter on the temperature distribution and stability characteristics of the micropolar boundary-layer flow over a semi-infinite flat plate. Particular attention is given to the differences between heat-generation and heat-absorption conditions. The discussion is based on the trends exhibited by the numerical profiles and stability curves presented in Figures 1–10.

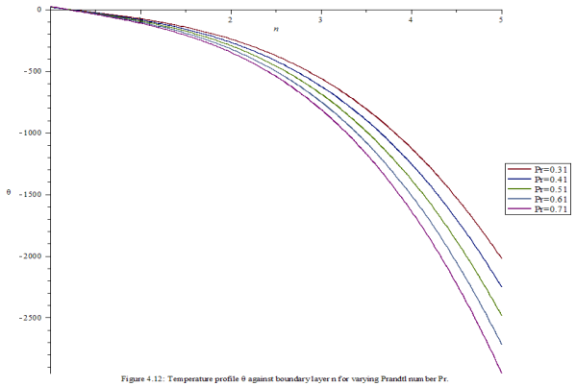


Figure 1: Effect of the Prandtl number (Pr) on the dimensionless temperature distribution (θ) as a function of the boundary-layer coordinate (n) under heat-generation conditions.

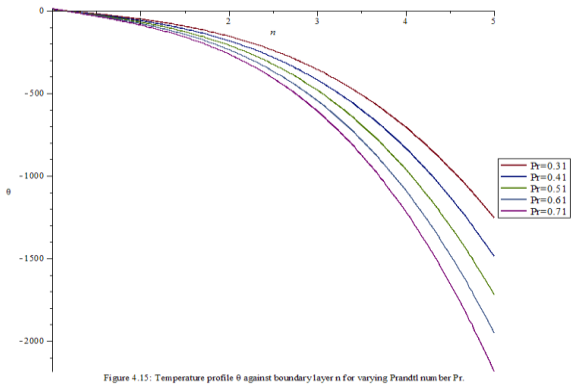


Figure 2: Effect of the Prandtl number (Pr) on the dimensionless temperature distribution (θ) as a function of the boundary-layer coordinate (n) under heat-absorption conditions.

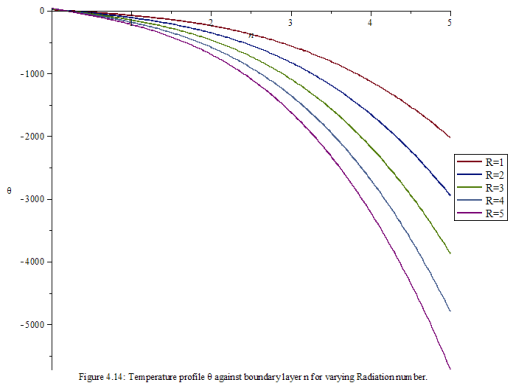


Figure 3: Effect of the thermal radiation parameter (R) on the dimensionless temperature distribution (θ) as a function of the boundary-layer coordinate (n) under heat-generation conditions.

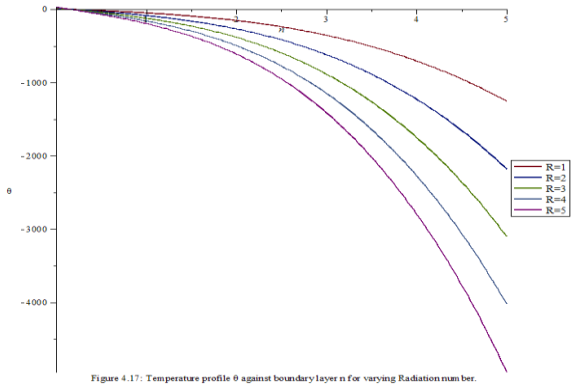


Figure 4: Effect of the thermal radiation parameter (R) on the dimensionless temperature distribution (θ) as a function of the boundary-layer coordinate (n) under heat-absorption conditions.

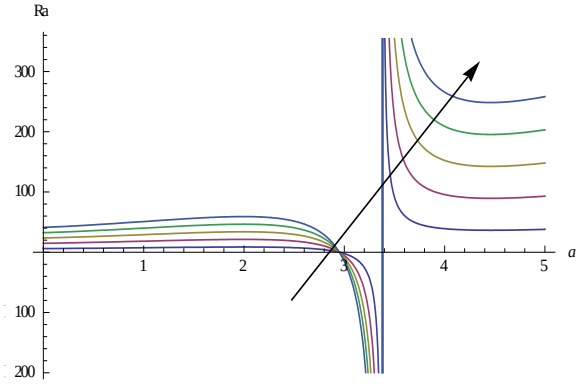


Figure 5: Variation of the Rayleigh number (Ra) with the disturbance wave number (a) for different values of the thermal radiation parameter (R) under heat-generation conditions when R = 1.0, 2.0, 3.0, 4.0, 5.0. For Q $\geq$ 1.5

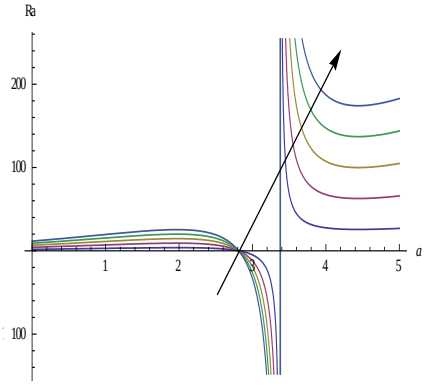


Figure 6: Variation of the Rayleigh number (Ra) with the disturbance wave number (a) for different values of the thermal radiation parameter (R) under heat-absorption conditions with R = 1.0, 2.0, 3.0, 4.0, 5.0. For Q $\leq$ 1.5

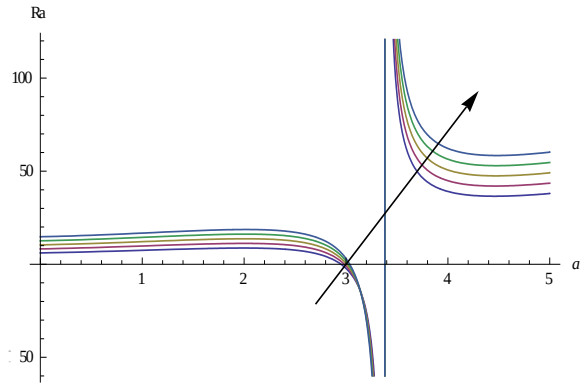


Figure 7: Variation of the Rayleigh number (Ra) with the disturbance wave number (a) for different values of the heat-function parameter (Q) under heat-generation conditions with Q =−1.5, -3.0, -4.5, -6.0, -7.5. For Q ≥1.5

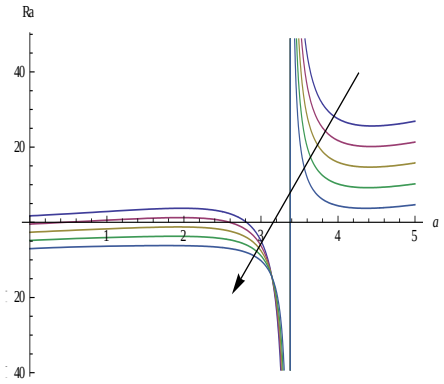


Figure 8: Variation of the Rayleigh number (Ra) with the disturbance wave number (a) for different values of the heat-function parameter (Q) under heat-absorption conditions with Q =−1.5, -3.0, -4.5, -6.0, -7.5. For Q ≤1.5

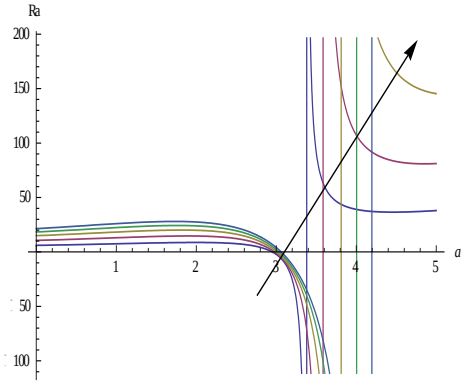


Figure 9: Variation of the Rayleigh number (Ra) with the disturbance wave number (a) for different values of the micropolar parameter (K) under heat-generation conditions with k = 0.95, 1.90, 2.85, 3.80, 4.75. For Q ≥ 1.5

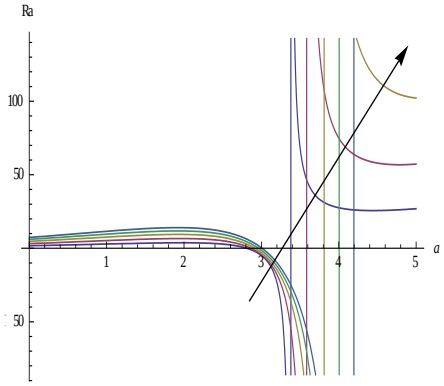


Figure 10: Variation of the Rayleigh number (Ra) with the disturbance wave number (a) for different values of the micropolar parameter (K) under heat-absorption conditions with k = 0.95, 1.90, 2.85, 3.80, 4.75. For Q ≤ 1.5

Temperature Distributions

Effect of the Prandtl Number

Figures 1 and 2 illustrate the influence of the Prandtl number ((Pr)) on the temperature distribution under heat-generation and heat-absorption conditions, respectively. The results show that increasing the Prandtl number enhances the temperature profile under heat generation but reduces it under heat absorption. This contrasting behaviour indicates that the thermal response of the fluid depends on the prevailing internal thermal condition.

The Prandtl number characterises the ratio of momentum diffusivity to thermal diffusivity and consequently influences thermal boundary-layer development. The observed increase in temperature under heat generation suggests that the thermal response is modified by the combined effects of thermal diffusion and internal heating, whereas the reduction under heat absorption reflects the influence of the internal thermal sink on the energy distribution within the boundary layer.

Effect of Thermal Radiation

Figures 3 and 4 present the temperature profiles for different values of the thermal radiation parameter ((R)) under heat-generation and heat-absorption conditions, respectively. The results indicate that increasing the radiation parameter reduces the dimensionless temperature distribution in both cases. This behaviour demonstrates the influence of radiative heat transfer on the energy distribution within the boundary layer. Although the direction of the temperature response is similar under both thermal conditions, the magnitude of the reduction may differ depending on the internal heat contribution.

Stability Characteristics

The stability characteristics are examined through the variation of the Rayleigh number ((Ra)) with the disturbance wave number ((a)). The resulting curves describe the relationship between the thermal driving force and the disturbance wave number under different parameter conditions. In accordance with the adopted neutral-stability formulation, the minimum of each neutral-stability curve defines the critical Rayleigh number ((Ra_c)) and its corresponding critical wave number ((a_c)), which characterise the onset of instability.

Effect of Thermal Radiation on Stability

Figures 5 and 6 show the stability curves for different values of the thermal radiation parameter under heat-generation and heat-absorption conditions, respectively. The plotted results indicate that increasing the radiation parameter is associated with an earlier onset of instability under both thermal conditions. This response suggests that thermal radiation influences stability through its contribution to the energy equation and the resulting temperature distribution. Since the thermal field is coupled to the momentum field through buoyancy effects, changes in temperature can modify the response of the basic flow to small disturbances. A quantitative assessment of this influence requires comparing the critical Rayleigh numbers and their corresponding wave numbers obtained from the minima of the neutral-stability curves.

Effect of the Heat-Function Parameter on Stability

Figures 7 and 8 illustrate the variation of the Rayleigh number with the disturbance wave number for different values of the heat-function parameter under heat-generation and heat-absorption conditions, respectively. The plotted results indicate that variations in the heat-function parameter are associated with an earlier onset of instability in both cases, based on the stability interpretation adopted in the present analysis. The heat-function parameter represents the contribution of internal heating or absorption to the energy balance. Its influence on stability is therefore related to modifications in the temperature field and the resulting interaction between thermal buoyancy and momentum transport. The interpretation of these trends must be consistent with the mathematical definition and sign convention adopted in the governing energy equation. In particular, positive and negative values should be associated with heat generation and heat absorption only as specified by the adopted formulation.

Effect of the Micropolar Parameter on Stability

Figures 9 and 10 present the stability curves for different values of the micropolar parameter (K) under heat-generation and heat-absorption conditions, respectively. The results indicate that increasing the micropolar parameter delays the onset of instability in both cases. The micropolar parameter accounts for the influence of fluid microstructure and governs the coupling between the linear momentum and microrotation fields. Changes in this parameter can therefore influence the basic-flow characteristics and the response of the fluid to imposed disturbances. The delayed onset of instability observed in Figures 9 and 10 suggests that the micropolar contribution has a stabilising influence within the range of parameter values investigated. The magnitude of this effect should be assessed by comparing the corresponding critical Rayleigh numbers and wave numbers.

Comparative Discussion

The temperature distributions and stability curves demonstrate that thermal and microstructural parameters influence the behaviour of micropolar boundary-layer flow over a semi-infinite flat plate. Increasing the Prandtl number enhances the temperature distribution under heat generation but reduces it under heat absorption, whereas

CONCLUSION

This study investigated the temperature distribution and stability characteristics of micropolar boundary-layer flow over a semi-infinite flat plate under thermal radiation and internal heat generation or absorption. The governing equations were solved using the collocation weighted residual method. The results revealed that increasing the Prandtl number enhances the temperature profile under heat generation but reduces it under heat absorption, while increasing thermal radiation lowers the temperature distribution in both cases. The stability curves further indicated that thermal radiation and variations in the heat-function parameter are associated with an earlier onset of instability, whereas increasing the micropolar parameter delays instability under both thermal conditions. These findings demonstrate the influence of thermal transport and fluid microstructure on the stability behaviour of the flow. However, the conclusions remain subject to the assumptions and parameter ranges of the model. Further studies should establish the critical Rayleigh numbers and corresponding wave numbers quantitatively and undertake independent numerical validation to strengthen the reliability of the predictions.

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