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Temporal Variability and Predictability of Radio Refractivity in Four Selected Locations of Locations in Nigeria

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ABSTRACT

Radio refractivity strongly influences atmospheric propagation and communication system reliability, yet its nonlinear dynamics remain underexplored in tropical West Africa. This study investigates the spatiotemporal variability and predictability of radio refractivity across four locations in Nigeria, using daily meteorological data of temperature, pressure and relative humidity from 1982–2022. Nonlinear methods were applied, including recurrence plots (RP), recurrence quantification analysis (RQA) and nonlinear indicators such as the Hurst exponent and Lyapunov exponent. Results reveal a strong climatic changes, Sudan/Sahelian states (Sokoto, Kano, Yobe and Borno) exhibit sparse RP structures, high recurrence rates (RR of 1.55–2.1%) and high determinism (DET of 0.23-0.30) and Hurst exponents of (0.71- 0.72). Across all the four locations, the Hurst exponent (HE > 0.5, which is > HT) exceeded the theoretical Hurst exponent (HT) baseline (HT = 0.5), indicating persistent long range dependence to initial conditions, meaning every location shows more persistence than the near random theoretical value of Hurst exponent. While, small but positive Lyapunov exponents (0.0040–0.0045 per day), defines the signature of weak chaos, radio refractivity is deterministic and future evolutions can be completely correlated to past behaviour and the predictability horizons of 250–222 days. The findings demonstrated that radio refractivity is persistent and predictable, though strongly modulated by climatic conditions. These result improve the understanding of tropical atmospheric variability and provide a foundation for refining radio propagation models and long term communication planning in the selected regions.

Keywords:
Radio Refractivity,
Recurrence Quantification Analysis,
Hurst Exponent Lyapunov Exponent.

INTRODUCTION

Radio refractivity is a fundamental parameter governing the propagation of electromagnetic waves in the atmosphere. It depends primarily on air temperature, pressure, and water vapour content, and directly influences terrestrial microwave links, satellite ground communication and radar performance. Refractivity is a crucial parameter in order to understand radio signal propagation and variations in refractivity can give rise to ducting, super-refraction, multipath fading and signal degradation, which are especially critical in regions where communication networks are expanding rapidly (Onuorah *et al.,* 2022). Temporal variations, driven by seasonal changes and weather patterns further complicate the prediction of radio wave behavior. For instance, during the rainy season, high humidity and frequent precipitation can substantially alter radio refractivity, while the dry season is often characterized by stable atmospheric conditions, leading to more predictable signal propagation (Emmanuel *et al*., 2023). Radio refractivity is typically highest in areas with significant moisture content, such as coastal and rainforest regions, due to the presence of water vapor, whereas drier areas tend to exhibit lower refractivity (Yusuf *et al*., 2022).

Most previous studies of refractivity variability in tropical environments such as (Tanko *et al*., 2025; Abimbola *et al.,* 2021 and Jibril *et al.,* 2021) have employed statistical or linear approaches, focusing on mean seasonal trends or regression based propagation models. While these methods provide valuable insights, they are limited in their ability to capture nonlinear behaviour and long term memory inherent in atmospheric processes. Refractivity is a product of coupled meteorological variables and is likely to exhibit complex dynamics including persistence, chaoticity and scale dependence (Ojo *et al.,* 2019). Recurrence analysis, which reconstructs the phase space of a time series to examine repeated states, provides a powerful framework for addressing these complexities. Recurrence plots (RP) offer a visual representation of state recurrence, while recurrence quantification analysis (RQA) provides measures such as recurrence rate, determinism, entropy and laminarity that quantify the structure of variability (Marwan *et al.,* 2007). Complexity and nonlinear trend in dynamical systems have been an issue in the area of communication due to his interference on radio signals. Recently, comparative study has been carried out extensively on the chaotic status of hourly wind speed data using RP and RQA (Adeniji *et al.,* 2018) and a research on the atmospheric chaoticity and complexity of radio refractivity derived from Akure station (Ogunsua *et al.,* 2018).

Nonlinear indicators such as the Hurst exponent and Lyapunov exponent further enhance this framework. The Hurst exponent characterises persistence, indicating whether variability tends to reinforce past trends or revert to the mean. The Lyapunov exponent quantifies divergence of trajectories in phase space, thereby providing insight into chaoticity and predictability horizons. Together, these methods allow for a holistic characterisation of refractivity variability across climatic zones. Nigeria, provides an ideal natural laboratory for examining nonlinear refractivity dynamics. Yet, no comprehensive long term study has applied recurrence based nonlinear analysis to refractivity across more diverse environment.

This study is aimed to analyse the temporal and spatial dynamics of radio refractivity across four locations in Nigeria using recurrence plots, quantify the complexity of refractivity variability with RQA, characterize the data based on Hurst and Lyapunov exponents values. The Lyapunov values were used to assess predictability horizons. By integrating these approaches, the study contributes both to theoretical understanding of atmospheric dynamics and to applied communication planning, where reliable knowledge of refractivity variability is crucial for infrastructure resilience.

MATERIALS AND METHODS

Study Area

Four locations were selected for analysis: Sokoto (Latitude 13.0501 and Longitude 5.2251**),** Kano (Latitude 12.0007 and Longitude 8.5266), Damaturu (Latitude 12.8771 and Longitude 10.4516) and Maiduguri (Latitude 11.8251 and Longitude 13.1024).

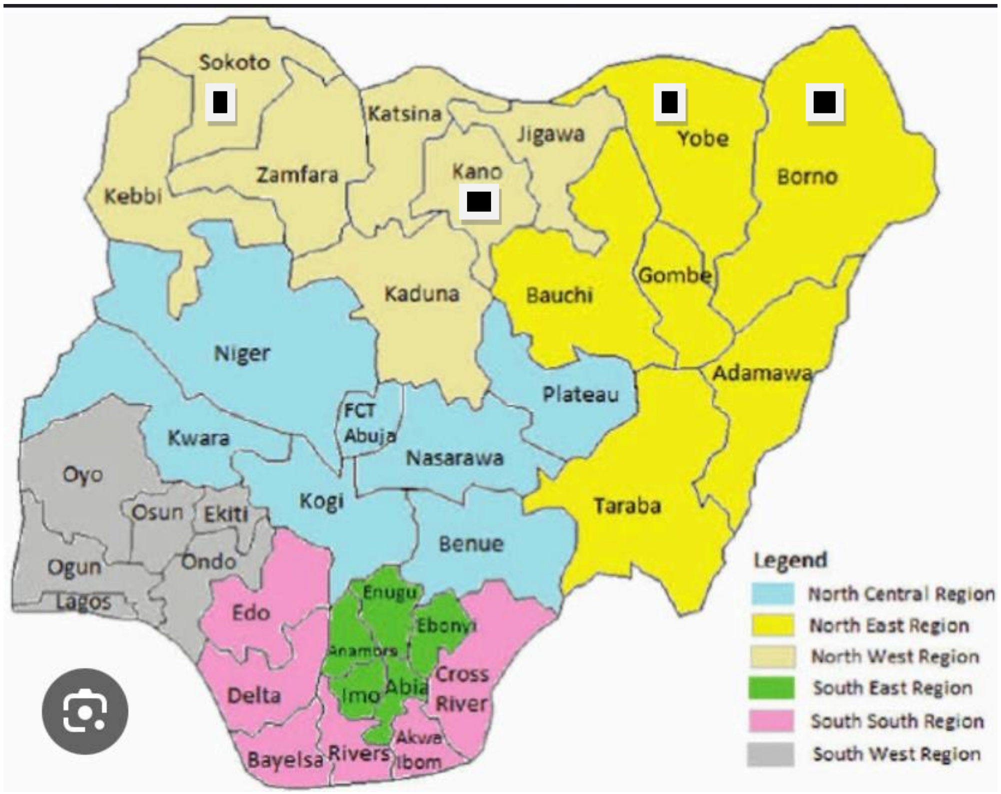

Figure 1: Map of Nigeria Showing the Four Selected Locations (Sokoto, Kano, Yobe and Borno)

Data Analysis

The daily meteorological data of temperature, humidity and pressure will be downloaded from the MERRA 2 of the National Aeronautical and Space Administration (NASA) for the 4 selected locations over a period of 41 years (1981-2022). The variables include surface temperature (T, in Kelvin), atmospheric pressure (P, in hPa) and relative humidity (RH, %). Water vapour pressure (e, hPa) was calculated from RH and saturation vapour pressure. Radio refractivity **N** (in N-units) was then computed using the procedure recommended by the International Telecommunication Union (ITU) for determining radio refractivity in terms of measured meteorological variables is given as ITU-R (2003);

$$N = 77.6\,\frac{P}{T} + 3.73 \times 10^5\,\frac{e}{T^2} \qquad (1)$$

where, T is the temperature (ºC + 273.15K) P is the atmospheric pressure and *e* is the water vapor pressure in hPa (hPa = 100Pa). $77.6\,\frac{P}{T}$ represent the dry air refractivity (dry term, N_{dry}) per unit pressure and temperature and it is large because of dry air has less polarization. $3.73 \times 10^5\,\frac{e}{T^2}$ represents the water vapor refractivity (wet term, N_{wet}) per unit vapor pressure and temperature and it is much larger because water vapor has a strong effect on refractivity due to its dipole moment.

The water vapor pressure is related to the relative humidity as Freeman (2007).

$$e = \frac{H.e_s}{100} \qquad (2)$$

The saturated vapor pressure (e_s) is given by Afullo *et al* (2004).

$$e_s = a \exp\left(\frac{bT}{T+c}\right) \qquad (3)$$

$$e_s = 6.112 \exp\left(\frac{17.50T}{T+240.97}\right) \qquad (4)$$

The coefficients a, b and c were represented by (ITU-R 2003), that for water were adopted as; a = 6.112, b = 17.502 and c = 240.97 and these are valid for -20 to +50 with an accuracy of +-0.20%. Equation (1), (2), (3) and (3) can be used to compute radio refractivity for all frequencies up to 100GHz, since the error is less than 0.5% and at sea level, the average value of N~315 was used (Freeman 2007; ITU-R, 2003 and Smith *et al.*, 1953).

Method

Recurrence Plots (RP)

This is a concept introduced by Eckmann *et al.* (1987) that focuses on the time dependent behavior of dynamical systems. It is a tool used to visualize the state-space dynamics. The RP gives the reader a first impression of the patterns of recurrences which will allow studying dynamical systems and their trajectories like periodic systems, stochastic/random systems and chaotic ones. A recurrence plot is mathematically expressed as;

$$D = \left[D_{i,j}\right] \quad \text{i, j=1, …..N} \qquad (5)$$

(N is the length of the data series)

$$D_{i,j} = \vec{x}_i - \vec{x}_j \qquad (6)$$

By applying threshold ε

$$R_{i,j} = \odot\left(\varepsilon - \mathrm{D_{i,j}}\right) \qquad (8)$$

where, $\odot(x)$ is the Heaviside function, given by;

$$\odot(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases} \qquad (9)$$

if $\vec{x}_i \approx \vec{x}_j$ then $R_{i,j} = 1$, if not, then $R_{i,j} = 0$. $R_{i,j}$ represents the recurrence matrix and ε is a predefined threshold that measures the distance between two neighbouring points. The variables $\vec{x}_i$ and $\vec{x}_j$ are the phase space points at time i and j respectively. The recurrence plots of different dynamical systems are shown in Figure 1 (A), (B) and (C) (Marwan *et. al.*, 2007).

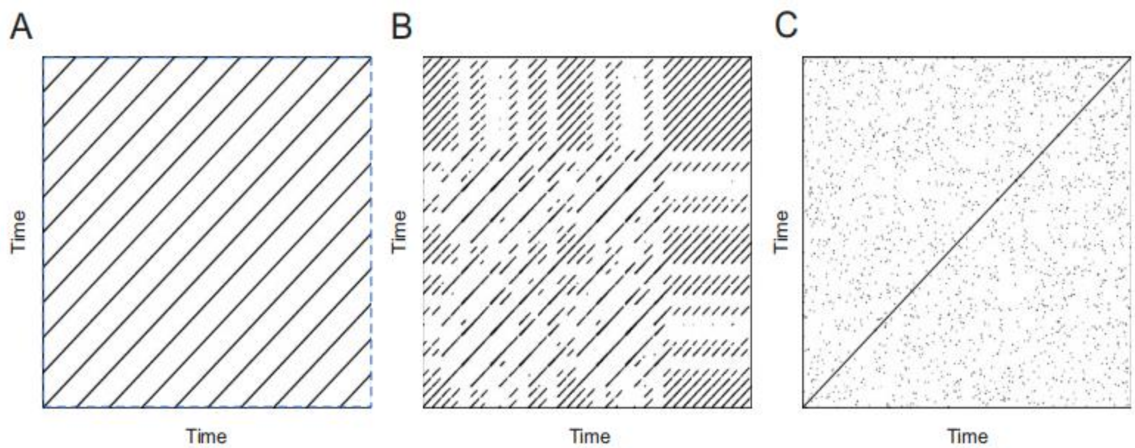

Figure 2: Recurrence Plots of Three Different Dynamical Systems

(A) Long and non-interrupted diagonals are related to periodic motion and the period of oscillation is equal to the vertical distance between these lines. (B) The diagonals are shorter it seems that the RP is related to the chaotic systems. (C) The RP with scattered points, no diagonal lines and so many single black points with erratic distribution is related to the uncorrelated stochastic signal.

Recurrence Quantification Analysis (RQA)

Recurrence Quantification Analysis (RQA) as introduced by Zbilut & Webber (1992) is a nonlinear analytical technique that quantifies the frequency and duration of recurrences in a system's phase space trajectory. RQA variables related to vertical structures can detect transitions within chaotic states. The following parameters of RQA, as detailed by (Marwan *et al*., 2007) were utilized to identify dynamical changes, complexity and stability of the time series within the recurrence plot.

Recurrence Rate (RR): The recurrence rate measures the density of recurrence points in the plot and is one of the simplest RQA variables. It is mathematically represented as (Marwan *et al*., 2007).

$$RR(\varepsilon) = \frac{1}{N^2}\sum_{i,j=1}^{N} R_{i,j}(\varepsilon) \quad (10)$$

where; $R_{i,j}(\varepsilon)$ = recurrence matrix, N = length of the time series.

Determinism (DET): This is the proportion of recurrence points that form diagonal lines, given by (Goswami, 2019).

$$DET = \frac{\sum_{l=l_{min}}^{N} lP(l)}{\sum_{l=1}^{N} lP(l)} \quad (11)$$

where P(l) represents the histogram of the diagonal line of lengths l. DET varies between 0 and 1, for periodic signals DET is close to 1 and stochastic signals approaching 0. However, lower DET results in chaotic systems while higher DET occurs in periodic behavior.

Laminarity (LAM): LAM quantifies the percentage of recurrence points forming vertical lines, indicating the presence of laminar states in the system. It is defined as (Ye *et al*., 2015).

$$LAM = \frac{\sum_{v=v_{min}}^{N} vP(v)}{\sum_{v=1}^{N} vP(v)} \quad (12)$$

where, P(v) is the histogram of the vertical line of lengths v. LAM decreases as more isolated recurrence points appear, rather than vertical structures.

Trapping Time (TT): Trapping time represents the average length of the vertical lines and is calculated as (Marwan *et al*., 2007).

$$TT = \frac{\sum_{v=v_{min}}^{N} vP(v)}{\sum_{v=v_{min}}^{N} P(v)} \quad (13)$$

Hurst Exponent (HE): The Hurst exponent is a tool used to assess the long-term memory of a time series. It is connected to the autocorrelations within the series and how quickly these correlations diminish as the time gap between value pairs grows (Hurst, 1951). The Hurst exponent was computed using the rescaled range (R/S) method to evaluate persistence. For a time series x(t), the rescaled range grows with length n according to.

$$\mathrm{E}\left(\frac{R(n)}{S(n)}\right) = \lim_{n\to\infty} X_n{}^{Hq} \quad (14)$$

where, R(n) is the range of the first n values, S(n) is the standard deviation, E[n] is the expected value, X_n is the number of data points in the time series. According Granero (2008) Hurst exponent can be used to characterize time series as; Values of H > 0.5 indicate persistence, H < 0.5 anti-persistence, and H = 0.5 random behavior.

Lyapunov Exponent (LE): The largest Lyapunov exponent was estimated to assess sensitivity to initial conditions and system predictability. A positive LE indicates divergence of nearby trajectories, reflecting chaotic behavior. Wolf *et al.* (1985) proposed a model equation to estimate the Lyapunov exponent from a time series.

$$\lambda = \lim_{t\to\infty} \frac{1}{t} \frac{\| D_{x(t)} \|}{\| D_{x(0)} \|} \quad (15)$$

where t is the time step, $D_{x(0)}$ is the initial condition at time t = 0, $D_{x(t)}$ is the separation distance between the i^{th} and j^{th} nearest neighbors at time t and ‖ is the Euclidean norm.

However, according to (Jolly *et al.,* 2015; Bensaïda, 2014; Siek *et al.*, 2011; Stefanski *et al.,* 2004; Wolf *et al.,* 1985) classified different dynamical systems based on the nature of the value of their Lyapunov exponent (LE). λ < 0, the system is said to exhibit asymptotic stability, the more the negative exponent the more the stability the system has. λ = 0, their orbit is a neutral fixed point or an eventually fixed point, the system is in sort of steady state mode and if λ > 0, the orbit of this dynamical system is unstable, chaotic and future evolutions are completely correlated to past behaviour.

Predictability of a Dynamical System: Lyapunov time, or error-folding time T, is the inverse of the largest Lyapunov exponent (λ) expressed in days (Stehlik, 2006). Given as;

$$T = \frac{1}{\lambda} \quad (16)$$

RESULTS AND DISCUSSION

Here, are the presentations and discussions of the results obtained from the analysis of temporal and spatial variations in radio refractivity across four selected locations in Nigeria.

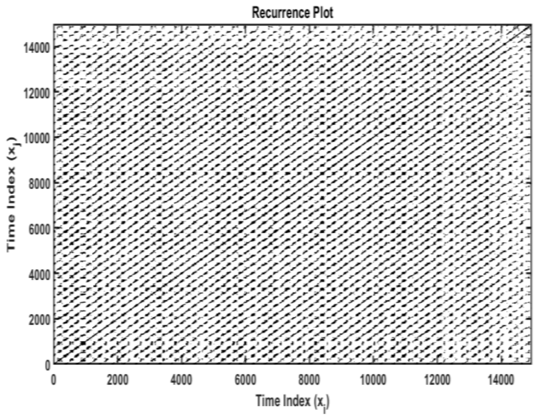

Figure 3: RP for Sokoto

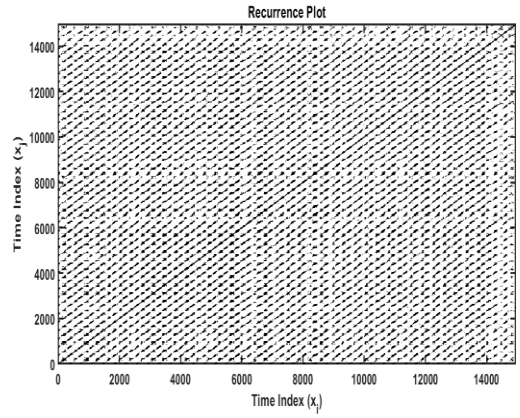

Figure 4: RP for Kano

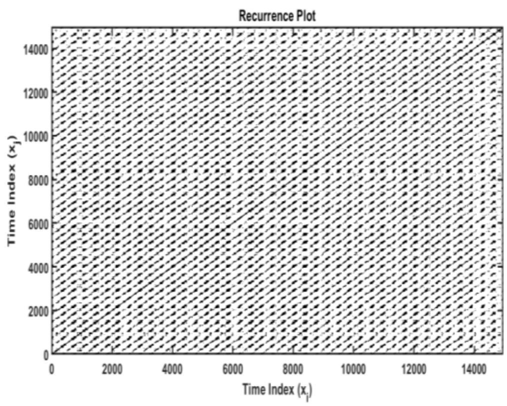

Figure 5: RP for Damaturu

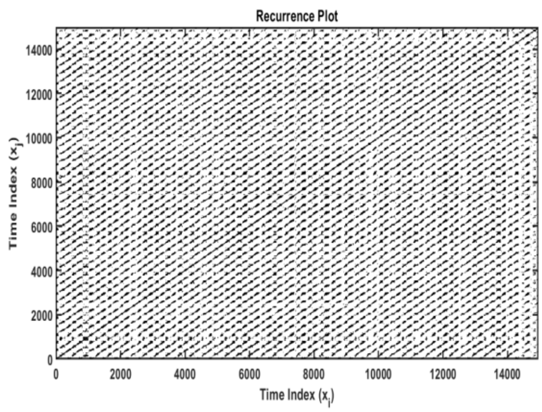

Figure 6: RP for Maiduguri

The mean radio refractivity (in N-units) are; Sokoto (315.236), Kano (308.242), Damaturu (305.592) and Maiduguri (311.579). Damaturu exhibit the lowest mean radio refractivity value while Sokoto have the highest value among the selected locations, this can be attributed to the fact that the Sokoto has less moisture content, as radio refractivity tends to be lower in a location with low humidity as according to a related research by (Tanko et al., 2025). And in another related research from (Tanko et al., 2019) has shown that radio refractivity has a strong positive relationship between humidity and a negative correlation with temperature.

From the RP plots, the RPs were found to be chaotic with the presence of very short diagonal lines, as discussed earlier from the classification of different dynamical systems (Figure 2). The chaotic behaviour can be attributed to the more unstable weather condition and erratic nature of rainfall in the locations, with short and heavy seasonal rain. From the RQA recurrence rate is high, DET is also high (20.16–23.82%), These indicate weak seasonal predictability, low frequency of stable atmospheric states and shorter persistence periods. this variability can be attributed to the fact that the weather in the locations are more unstable throughout a year, irregular wet season and frequent dust intrusions (Harmattan).

Results from the nonlinear analysis (LE and HE) shows that; all the four locations have positive LE ($\lambda > 0$), indicating that the system is unstable and chaotic**,** and future evolutions are completely correlated to past behaviour. The LE confirm the presence of weak chaotic dynamics, indicating that short term variability is significant and long term stability is limited.

The HE are all > the theoretical exponent value of 0.5 (that is HE ranges from 0.71-0.72), meaning every station shows more persistence than the near random theoretical value of Hurst exponent of 0.5, that a high/low value in the series will probably be followed by another high/low value. And the predictability window of about 221-249 days (7-8 months < a year).

Table 1: Radio Refractivity RQA Values

Location	RR (%)	DET	LAM	TT
Sokoto	1.5559	0.2382	0.4295	0.0436
Kano	1.5172	0.2657	0.4528	0.0448
Damaturu	2.139	0.3098	0.511	0.0449
Maiduguri	2.0175	0.3005	0.4838	0.0459

Table 2: Radio Refractivity Nonlinear Statistical Values

Location	LE (day^{-1})	Lyapunov time T (days)	≈ months	HRS	HT
Sokoto	0.004255	235.	7.7	0.723	0.542
Kano	0.004511	221.7	7.3	0.727	0.542
Damaturu	0.004013	249.2	8.2	0.715	0.542
Maiduguri	0.004251	235.2	7.7	0.71.	0.542

CONCLUSION

This study has characterized the temporal variability of radio refractivity across four locations in Nigeria using recurrence plots, recurrence quantification analysis and nonlinear tools (Hurst and Lyapunov exponents). The results reveal a clear climatic gradient with sparse recurrence, strong seasonality and shorter predictability horizons (~200 days). Hurst exponents confirm persistence in all regions, while Lyapunov exponents indicate weak chaoticity with finite prediction verdicts. These findings emphasize that refractivity in these four selected locations is chaotic and climate driven. The results provided valuable guidance for communication system design and planning. networks design in the locations require adaptive short term strategies to manage strong seasonal variability. This research advances understanding of refractivity dynamics and provides a nonlinear systems framework for improving radio communication reliability. And for further research, recurrence based analysis should be expanded to higher frequency microwave and millimetre wave bands, which are increasingly important for 5G and beyond and incorporate satellite based remote sensing data to complement ground measurements and enhance spatial mapping of refractivity dynamics.

REFERENCES

Abimbola, O.J., Bada, S.O., Folaiye A.O., Suam, Y. M., Otto, M.S and Muhammad, S (2021). Estimation of Radio Refractivity from Satellite Derived Meteorological Data over a Decade for West Africa. *Scientific African Journal.* Pg14. https://doi.org/10.1016/j.sciaf.2021.e01054

Afullo, T.J, & Odedina P.K. (2006) On the k-factor Distribution and Diffraction Fading for Southern Africa. *Southern African Institute of Electrical Engineers Resaerch Journal.* 97(2), 172-181.

Eckmann, J.P., Kamphorst, S., & Ruelle, D. (1987), Recurrence plots of dynamical systems, *Eutrophys, Lett* 4, 973-977. https://doi.org/10.1209/0295-5075/4/9/004

Goswami, B. (2019). A Brief Introduction to Nonlinear Time Series Analysis and Recurrence Plots. *Vibration 2*. 332–368; https://doi.org/10.3390/vibration2040021. www.mdpi.com/journal/vibration.

Granero Sánchez, J.E. Trinidad Segovia, J. García Pérez. Physica A 387 (2008). Some comments on Hurst exponent and the long memory processes on capital markets. M.A. 5543–5551.

Hurst, H (1951). Long Term Storage Capacity of Reservoirs, Transactions of the American Society of Civil Engineers 6, Page 770–799.

International Telecommunication Union Recommendation, ITU-R.(2003) The Radio Refractive Index: Its formula and refractivity data. Radio communication Study Group III ;453-459.

Jas and Adeniji (2021). Characterization and investigation of nonlinear behaviour of radio refractivity during the rainy and dry seasons in the coastal region of Nigeria. https://dx.doi.org/10.4314/jasem.V25:7:8

Jibril Y.A. Muhammad B.A., Muhammad A.W., Ndanusa, B., Mohammed, I.K., Abdullah, S.A., Umar, S and Danjuma A.B (2021). Varation in Meteorological Parameters on the Tropospheric Radio Refractivity Over Lapai, Niger State Nigeria. *Lapai Journal of Science and Technology* 7(1).

Jolly Watt M, Mark. (2015). *Journal of Nature Communicatio*n, article number: 7537

Marwan, N., Romano, M.C., Thel, M., Kuths, J. (2007). Recurrence Plots for the Analysis of Complex Systems. Physics Reports, 438(5-6), 237-329. (www.recurrence-plot.tk)

Marwan, N., Romano, M.C., Thel, M., Kuths, J. (2007). Recurrence Plots for the Analysis of Complex Systems.

Physics Reports, 438(5-6), 237-329. (www.recurrence-plot.tk)

Ogunsua. B.O, Joseph Sunday Ojo, and A.T Adediji (2018). Atmospheric Chaoticity and Complexity from Radio Refractivity Derived From Akure Station, Nigeria. (*Elsevier BV), advances in space research.* Vol.62, Iss: 7. Pp 1690-1701.

Ojo, J.S., Adelakun A.O., Edward O.V (2019). Comparative Study on Radio Refractivity Gradient in the Troposphere sing Chaotic Quantifiers. *Elsevier: Heliyon* Vol.5, Issue 8 https://doi.org/10.1016/J.Heliyon.2019.E02083.

Onuorah, L.O., Agbo, G.A and Ibanga, E.A (2019). Day-to-Day Seasonal Variations of Tropospheric Radio Refractivity in Lagos, South Western Nigeria. *International Research Journal of Innovations in Engineering and Technology* (IRJIET). 3(12): 36-41.

Siek, M. and Solomatine, D.P., (2011). Optimized dynamic ensembles of multiple chaotic models in predicting storm surges. Journal of Coastal Research, SI 64 (Proceedings of the 11th International Coastal Symposium), 1184 – 1188. Szczecin, Poland, ISSN 0749-0208

Smith, E.K., & Weintraub, S. (1953). The constants in the equation for atmospheric refractive index at radio frequencies. *proceedings of the I.R.E conference*, 4(8), 1035-1037.

Stefanski A, Dabrowski, (2004) Evaluation of the largest Lyapunov exponent in dynamical systems with time delay. *Chaos Solutions & Fractals* 23(5):1651 1659 http://dx.doi.org/10.1016/j.chaos.2004.06.051

Stehlík. J, (2006). Searching for Chaos in Rainfall and Temperature Records – a Nonlinear Analysis of Time Series from an Experimental Basin. *Na Sabatce*, 17(143), 6.

Tanko M, M. U. Sarki, U. Rilwan and I. S. Adeshina (2025). Tropospheric Radio Refractivity Mapping of Nigeria from satellite Data. *Nexus of Advanced Environmental Research.* 1(2025) pg 1-9. https://noaerjournal.com.

Tanko, M. M., Sarki, M.U and Bilya, M.A (2019). Seasonal Variation of Radio Refractivity of Some Selected Stations in Northern Nigeria. *Current Journal of Applied Science And Technology.* 32(1): 1-12. https://doi.org/10.9734/cjast/2019/45326

Wolf A, Swift J.B, Swinney H.L, Vastano J.A (1985) Determining Lyapunov exponents from a time series. *Physics Nonlinear Phenomenon*. 16(3):285–317. https://doi.org/10.1093/rof/rfs003

Ye H, E. R. Deyle, L. J. Gilarranz & G. Sugihara (2015). Distinguishing time-delayed causal interactions using convergent cross mapping. *Scientific Reports* vol.5, no. 1, article 14750.

Yusuf, Y., Aboloma, C. A and Oloke, O.R (2022). Monthly Average Estimation of Radio Refractivity of Ado-Ekiti South-Western Nigeria. *Journal of Engineering and Earth Sciences.* 15(1): Pg 1-8.

Zbilut, J and Webber J.C. (1992),’’Embeddings and delays as derived from quantification of recurrence plots,’’ Physics Letters A 171, 199-203